University of East Anglia

STUDENT SUPPORT SERVICE

Solving Quadratic Equations by Factorisation

This guide looks at quadratic equations and gives a basic technique for their solution. The solutions are described graphically as particular points on the corresponding quadratic function.

Introduction

Before reading this guide you should read the study guide: *Quadratic Functions* to familiarise yourself with the language used here. You should also read the study guide: *Factorising Quadratic Expressions* to understand the technique of factorisation.

Every quadratic function takes the form:

$$y = ax^2 + bx + c$$

and this is known as the **general form of the quadratic function**. When you solve a quadratic equation, what you are doing is finding the points where the quadratic function crosses the x-axis. Crucially, **at every point on the *x*-axis, *y* takes the value of 0**. Setting y equal to zero in the general quadratic function leads to the **general quadratic equation.** By setting y equal to zero in the function above you get:

$$ax^2 + bx + c = 0$$

The solutions of this equation give the points at which the graph of the corresponding function crosses the x-axis. In mathematics the solutions have a special name and are known as the **roots** of the equation. There are a variety of methods for solving quadratic equations (see the study guides: *Solving Quadratic Equations Using the Quadratic Formula* and *Completing the Square*). The simplest and quickest method of finding the roots is by factorising the quadratic expression on the left-hand side of the equals sign.

Quadratic equations with no constant term

Quadratic equations with no constant term are straightforward to solve. In other words if the number represented by *c* in the general equation is zero you have:

$$ax^2 + bx = 0$$

There is a common factor of *x* in each term on the left-hand side of the equation and so the equation can be factorised using simple factorisation (see study guide: *Simple Factorisation*). So the equation becomes:

$$x(ax + b) = 0$$

Importantly, you have *two* things multiplied together on the left-hand side of the equation (x and $ax + b$). In order for $x(ax + b)$ to be equal to zero either *x* or $ax + b$ must equal zero. You can write this mathematically as:

$$x = 0 \qquad \text{or} \qquad ax + b = 0$$

the first equation gives a root immediately as $x = 0$. The second equation gives a second root by rearranging $ax + b = 0$ for *x*. After the rearrangement you find that:

$$x = -\frac{b}{a}$$

So for a general quadratic equation with $c = 0$ the roots are $x = 0$ and $x = -b/a$.
If you are given *a* and *b* in a particular question you can write the two roots down immediately.

Example: Where does the function $y = 6x^2 - 5x$ cross the *x*-axis?

This question is exactly the same as "solve $6x^2 - 5x = 0$" (note that $a = 6$ and $b = -5$). By simple factorisation of the common factor *x* you can write the equation as:

$$x(6x - 5) = 0$$

and so $x = 0$ or $6x - 5 = 0$. You can find the roots by either rearrangement or by using the formula above. The roots, and hence the points where the function crosses the *x*-axis are $x = 0$ and $x = \frac{5}{6}$.

Quadratic equations which can be factorised-by-inspection

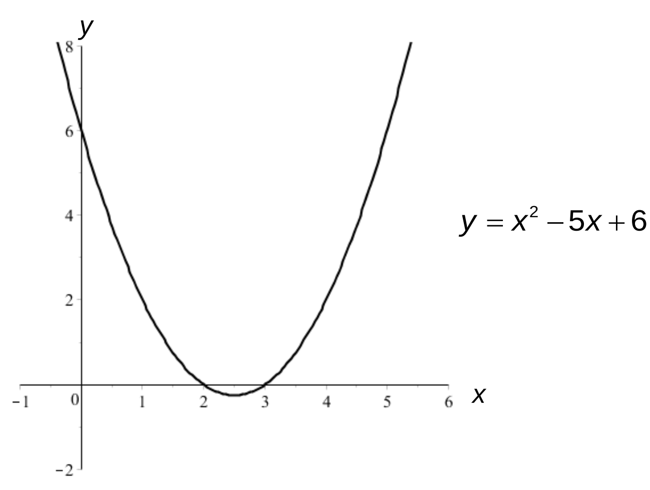
If you decide that the quadratic equation you are solving can be factorised-by-inspection then you may use that method. It is extremely useful to identify these equations as factorisation-by-inspection is quicker and easier than other methods for finding roots. As your experience of factorising-by-inspection increases you may be able to factorise more complicated quadratic expressions.

Example: Find the roots of $y = x^2 - 5x + 6$.

This is the same as the question "solve $x^2 - 5x + 6 = 0$". This equation can be factorised-by-inspection to give:

$$(x-2)(x-3) = 0$$

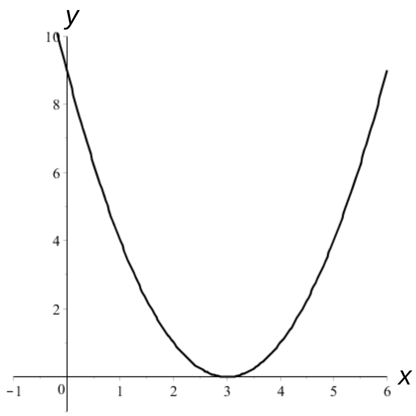
You have two terms on the left-hand side of the equation which, when multiplied together equal 0 and so either $(x-2) = 0$ or $(x-3) = 0$. Rearranging the first equation for *x* gives $x = 2$ and rearranging the second equation for *x* gives $x = 3$. The two roots are $x = 2$ and $x = 3$. On the function graph the roots represent points where the function crosses the *x*-axis, here at $x = 2$ and $x = 3$, as can be seen in the graph below:



You may find that you have identical brackets after factorisation-by-inspection, this represents a function which touches the *x*-axis at a single point. In this case both the roots will be equal to the same number.

Example: Where does the function $y = x^2 - 6x + 9$ cross the *x*-axis?

By factorising the corresponding quadratic equation $x^2 - 6x + 9 = 0$ you get $(x-3)(x-3) = 0$ which is more simply written as $(x-3)^2 = 0$. So the root is $x = 3$ and the quadratic function just touches the *x*-axis at $x = 3$ as shown in the graph below.



$y = x^2 - 6x + 9$

Want to know more?

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