Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Algebra

Factorising Quadratic Expressions

This guide introduces quadratic expressions and gives a simple technique to aid in their factorisation

Introduction

Quadratic expressions are very common in mathematics and it is useful if you can identify them and then factorise them. For example, being able to factorise a quadratic *expression* is extremely useful when solving quadratic *equations*. Factorisation of quadratic equations leads to their **roots** and there are many methods to find these roots. However factorisation is the simplest and quickest and it is beneficial to practise this skill. Here you will concentrate on the technique of **factorisation-by-inspection** (or **factorisation-by-eye**).

What is a quadratic expression?

Technically a quadratic expression is a **polynomial expression of order two**, but what does this mean? Every quadratic expression fits the pattern:

$$ax^2 + bx + c$$

where a, b and c are numbers (called **coefficients**) and x is a **variable**. The coefficient a can take any positive or negative value except for 0, b and c can take any value at all. You can see that a quadratic expression has a highest power of x of 2, which is why it is of order 2. It may, but not necessarily, contain two other terms: one dependent on x and a constant c at the end. The term quadratic comes from the Latin word for square: *quadratus*.

Example: Which of these expressions are quadratic in nature?

(a) $3x^2 + 4x - 5$ (b) $-x^2 - \frac{1}{4}$ (c) $5 - 3x + x^2$ (d) $x + 4$

(e) $x^{-2} + x^2$ (f) $x^3 + x^2 - 2x - 9$ (g) $4x^2 - 3x$.

(a) A quadratic expression with $a = 3$, $b = 4$ and $c = -5$.

(b) A quadratic expression with $a = -1$, $b = 0$ and $c = -\frac{1}{4}$.

(c) A quadratic expression with $a = 1$, $b = -3$ and $c = 5$, term order is unimportant.

(d) Not a quadratic expression as there is no x^2 term; this is a linear expression.

(e) Not a quadratic expression due to the x^{-2} term.

(f) Not a quadratic due to the x^3 term.

(g) A quadratic expression with $a = 4$, $b = -3$ and $c = 0$.

Factorising quadratic expressions

Before you learn how to factorise certain quadratic expressions it is important not to forget to account for any common factors that may be part of the expression. These need to be factored out before you begin the analysis outlined below. Your first thought should always be "are there any common factors?", if so you can factorise them out using the method of simple factorisation; see study guide: *Simple Factorisation*. You will concentrate on quadratic expressions without any common factors here.

Factorisation introduces brackets into an expression or equation and in order to set up the technique let's expand the brackets $(x+m)(x+n)$ where *m* and *n* are numbers:

$$\begin{aligned}(x+m)(x+n) &= x^2 + mx + nx + mn \\ &= x^2 + (m+n)x + mn\end{aligned}$$

where the two middle terms have been factorised themselves. If you find this difficult see the study guide: *Opening Brackets*. Can you see that the expanded form is a quadratic expression? Specifically, if the number multiplied by x^2 is 1, the number multiplied by *x* is $m+n$ and the constant is *m* multiplied by *n*. Thinking of this process in reverse offers a strategy to factorise quadratic expressions which fit this specific pattern.

Example: Factorise $x^2 + 3x + 2$.

This quadratic expression fits the pattern of $x^2 + (m+n)x + mn$ where *m* multiplied by *n* is equal to +2 and $m+n$ is equal to +3. You need to find the answer to the question "which two numbers are multiplied together to give 2 and added together to make 3?". Can you see that the numbers must be 1 and 2 as $1 \times 2 = 2$ and $1 + 2 = 3$. These are *m* and *n*. Substituting these results into $(x+m)(x+n)$ gives;

$$x^2 + 3x + 2 = (x+1)(x+2)$$

which can be easily checked by opening the brackets. The technique of factorisation-by-eye is precisely this, asking the question “which two numbers do I need to multiply together to give … and then add together to give …?”.

Example: Factorise $x^2 - x - 12$.

This is more difficult. Which two numbers multiply together to give -12 and add together give -1? If you cannot deduce them you need another strategy. Create a table to help you. Draw a table with two columns, in the left-hand column list all the whole numbers which multiply to give -12. A good way of creating this list is to start with 1, then 2, then 3 and so on. Once you have a complete list, in the right-hand column add the numbers. If one of the sums is equal to the number you require then you have found the two numbers! The table below is for the question above.

multiplied to make -12	**added**
$+1 \times -12$	$+1 + -12 = -11$
$+2 \times -6$	$+2 + -6 = -4$
$+3 \times -4$	$+3 + -4 = -1$
$+4 \times -3$	$+4 + -3 = +1$
$+6 \times -2$	$+6 + -2 = +4$
$+12 \times -1$	$+12 + -1 = +11$

In this case can you see that the numbers needed are $+3$ and -4? They are circled to help you. So $x^2 - x - 12 = (x+3)(x-4)$. Importantly the table above gives **every** possible whole number for $m+n$ when *m* multiplied by *n* equals -12. These are specifically -11, -4, -1, $+1$, $+4$ and $+11$. This allows you to easily see whether a quadratic expression can be factorised using this method or not. So $x^2 - 2x - 12$ cannot be factorised by this method as -2 does not appear in the right-hand list above. Take care when the numbers you need are of different signs, make sure that you assign the correct sign for each number.

Example: Factorise $x^2 - 9x + 14$.

On first inspection this seems impossible as you have 1×14 and 2×7 giving 14, but neither $1 + 14$ nor $2 + 7$ equal -9.

However something that is often forgotten is that two negative numbers multiply to give a positive number so you also have -1×-14 and -2×-7 giving 14.

As $-2 + -7 = -9$ then $x^2 - 9x + 14 = (x-2)(x-7)$.

The difference of two squares

A special case of a quadratic expression is the **difference of two squares**. The general quadratic expression for the difference of two squares is $x^2 - m^2$ and can be factorised to give:

$$x^2 - m^2 = (x + m)(x - m)$$

Importantly the left-hand side contains a subtraction *not* an addition. Identifying the quadratic expression $x^2 - m^2$ allows the factorised form to be written down immediately. (Of course if you don't realise you have the difference of two squares, the previous method still applies where the addition of the two numbers is equal to 0.)

Example: Factorise $x^2 - 36$.

Notice that 36 is a square number (6^2) so you have the difference of two squares. Using the formula above with $m = 6$, you find:

$$x^2 - 36 = x^2 - 6^2 = (x + 6)(x - 6)$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761

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