Learning Enhancement Team

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Model Answers: Factorising Quadratic Expressions

Factorising Quadratic Expressions study guide

(a) $x^2+2x+1=(x+1)(x+1)$

To factorise this quadratic expression you need to find two numbers that multiply together to give the constant $+1$ and also add to give the coefficient of x which is $+2$.

Multiplied to make $+1$	Added
$+1\times+1$	$+2$
-1×-1	-2

From the table you can see that the numbers needed are $+1$ and $+1$. So:

$$x^2+2x+1=(x+1)(x+1)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(b) $x^2+8x+7=(x+1)(x+7)$

To factorise this quadratic expression you need to find two numbers that multiply together to give the constant $+7$ and also add to give the coefficient of x which is $+8$.

Multiplied to make $+1$	Added
$+1\times+7$	$+8$
-1×-7	-8

From the table you can see that the numbers needed are $+1$ and $+7$. So:

$$x^2+8x+7=(x+1)(x+7)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(c) $x^2 + 8x + 12 = (x + 2)(x + 6)$

To factorise this quadratic expression you need to find two numbers that multiply together to give the constant $+12$ and also add to give the coefficient of x which is $+8$.

Multiplied to make $+12$	Added
$+1 \times +12$	$+13$
$+2 \times +6$	$+8$
$+3 \times +4$	$+7$
-1×-12	-13
-2×-6	-8
-3×-4	-7

From the table you can see that the numbers needed are $+2$ and $+6$. So:

$$x^2 + 8x + 12 = (x + 2)(x + 6)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

The table in part (c) can be used to answer parts (d) to (h):

(d) From row 3 $x^2 + 7x + 12 = (x + 3)(x + 4)$

(e) From row 1 $x^2 + 13x + 12 = (x + 1)(x + 12)$

(f) From row 6 $x^2 - 7x + 12 = (x - 3)(x - 4)$

(g) From row 5 $x^2 - 8x + 12 = (x - 2)(x - 6)$

(h) From row 4 $x^2 - 13x + 12 = (x - 1)(x - 12)$

You can check all of these answers by multiplying out the brackets (see study guide: *Opening Brackets*).

2.

(a) $x^2 - x - 12 = (x + 3)(x - 4)$

To factorise this quadratic expression you need to find two numbers that multiply together to give the constant -12 and also add to give the coefficient of x which is -1.

Multiplied to make -12	Added
$+1 \times -12$	-11
$+2 \times -6$	-4
$+3 \times -4$	-1
$+4 \times -3$	$+1$
$+6 \times -2$	$+4$
$+12 \times -1$	$+11$

From the table you can see that the numbers needed are $+3$ and -4. So:

$$x^2 - x - 12 = (x + 3)(x - 4)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(b) $x^2 + x - 12 = (x + 4)(x - 3)$

You can use the table in question 2.(a) but this time you need two numbers which multiply together to give -12 and also add to give $+1$. From row 4 of the table you can see that the numbers needed are $+4$ and -3. So:

$$x^2 + x - 12 = (x + 4)(x - 3)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(c) $x^2 - 3x - 10 = (x + 2)(x - 5)$

To factorise this quadratic expression you need to find two numbers that multiply together to give the constant -10 and also add to give the coefficient of x which is -3.

Multiplied to make -10	Added
$+1 \times -10$	-9
$+2 \times -5$	-3
$+5 \times -2$	$+3$
$+10 \times -1$	$+9$

From the table you can see that the numbers needed are $+2$ and -5. So:

$$x^2 - 3x - 10 = (x + 2)(x - 5)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(d) $x^2 + 3x - 10 = (x + 5)(x - 2)$

You can use the table in question 2.(c) but this time you need two numbers which multiply together to give -10 and also add to give $+3$. From row 3 of the table you can see that the numbers needed are $+5$ and -2. So:

$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

3.

(a) $6x^2 - 3x = 3x(2x - 1)$

Although this is a quadratic expression, as there are common factors you can factorise it using simple factorisation (see study guide: *Simple Factorisation*). There is a common factor of $3x$ in $6x^2 - 3x$, therefore:

$$6x^2 - 3x = 3x(2x - 1)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(b) $x^2 - 100 = (x + 10)(x - 10)$

This quadratic expression is an example of the **difference of two squares**. The difference of two squares is method which factorises quadratic expressions of the form $x^2 - m^2$ as $(x + m)(x - m)$. In this example $m = 10$ and so:

$$x^2 - 100 = (x + 10)(x - 10)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

You can also factorise this equation by finding two numbers that multiply together to give the constant -100 and also add to give the coefficient of x which is 0.

(c) $x^2 - 1 = (x+1)(x-1)$

This quadratic expression is another example of the **difference of two squares**. The difference of two squares is method which factorises quadratic expressions of the form $x^2 - m^2$ as $(x+m)(x-m)$. In this example $m = 1$ and so:

$$x^2 - 1 = (x+1)(x-1)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

You can also factorise this equation by finding two numbers that multiply together to give the constant -1 and also add to give the coefficient of x which is 0.

(d) $4x^2 - 36 = (2x+6)(2x-6)$

This quadratic expression is a more complicated example of the **difference of two squares**. The difference of two squares is method which factorises quadratic expressions of the form $x^2 - m^2$ as $(x+m)(x-m)$. In this example $m = 6$ and the first term is $(2x)^2$ so:

$$4x^2 - 36 = (2x+6)(2x-6)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(e) $7x^2 - 2 = \left(\sqrt{7}x + \sqrt{2}\right)\left(\sqrt{7}x - \sqrt{2}\right)$

This quadratic expression is a more complicated example of the **difference of two squares**. The difference of two squares is method which factorises quadratic expressions of the form $x^2 - m^2$ as $(x+m)(x-m)$. In this example $m = \sqrt{2}$ as $\left(\sqrt{2}\right)^2 = 2$ and the first term is $\left(\sqrt{7}x\right)^2$ so:

$$7x^2 - 2 = \left(\sqrt{7}x + \sqrt{2}\right)\left(\sqrt{7}x - \sqrt{2}\right)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

(f) $\frac{x^2}{4}-1=\left(\frac{x}{2}+1\right)\left(\frac{x}{2}-1\right)$

This quadratic expression is a more complicated example of the **difference of two squares**. The difference of two squares is method which factorises quadratic expressions of the form x^2-m^2 as $(x+m)(x-m)$. In this example $m=1$ and the first term is $(x/2)^2$ so:

$$\frac{x^2}{4}-1=\left(\frac{x}{2}+1\right)\left(\frac{x}{2}-1\right)$$

You can check your answer by multiplying out the brackets (see study guide: *Opening Brackets*).

4. What whole number values of *a* mean that you can factorise the quadratic expression $x^2+ax+16$?

To find valid values for *a* you need to add together any two numbers that multiply together to give the constant $+16$:

Multiplied to make $+16$	Added
$+1\times+16$	$+17$
$+2\times+8$	$+10$
$+4\times+4$	$+8$
-4×-4	-8
-2×-8	-10
-1×-16	-17

From the table you can see that valid choices for *a* are 17, 10, 8, -8, -10 and -17. These give the quadratic expressions

$x^2+17x+16=(x+1)(x+16)$ $\quad$ $x^2-17x+16=(x-1)(x-16)$

$x^2+10x+16=(x+2)(x+8)$ $\quad$ $x^2-10x+16=(x-2)(x-8)$

$x^2+8x+16=(x+4)(x+4)=(x+4)^2$ $\quad$ $x^2-8x+16=(x-4)(x-4)=(x-4)^2$

You can check all these answers by multiplying out the brackets (see study guide: *Opening Brackets*).