Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Steps into Algebra

Quadratic Functions

This guide introduces the general form of a quadratic function and also describes their corresponding graphs.

Introduction

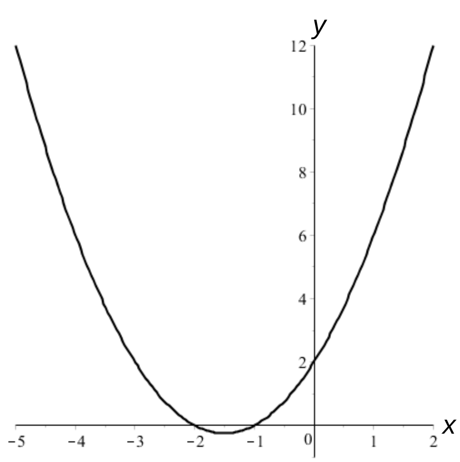
Every quadratic function takes the form:

$$y = ax^2 + bx + c$$

where *a*, *b* and *c* are numbers (called coefficients) and *x* is a variable. The coefficient *a* can take any positive or negative value except for 0; *b* and *c* can take any value at all.

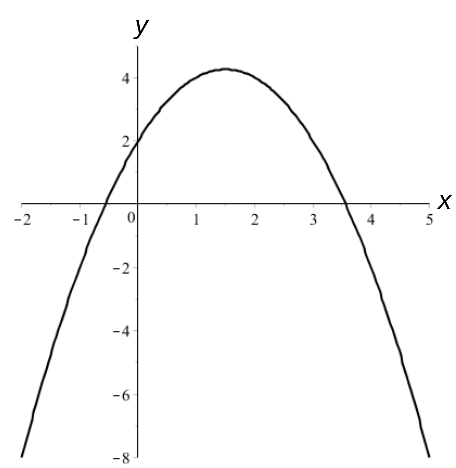
The graph of **every** quadratic function is a curve called a **parabola**. A parabola is a special, symmetrical curve which is one of the **conic sections**; i.e. it is formed by cutting a cone in a specific way.

For example $y = x^2 + 3x + 2$ and $y = -x^2 + 3x + 2$ are quadratic functions with their corresponding graphs given below:



$y = x^2 + 3x + 2$

$a = 1,\ b = 3$ and $c = 2$



$y = -x^2 + 3x + 2$

$a = -1,\ b = 3$ and $c = 2$

The effect of different values of *a*, *b* and *c*.

Different values of *a*, *b* and *c* in the general quadratic function control the positioning and steepness of the corresponding graph.

a: If *a* is positive the graph is a valley shape, if *a* is negative it is a hill shape. The larger the magnitude of *a*, then the steeper the graph.

b: The value of $-\frac{b}{2a}$ controls the **axis of symmetry** of the parabola. The axis of symmetry is the mirror plane of the graph and moves the graph left and right dependent on the values of *a* and *b*. The value of *b* alone controls the slope of the graph as it crosses the *y*-axis.

c: Increasing values of *c* move the graph up and decreasing values of *c* move the graph down.

Significant points on a parabola

There are some points on the graph of a quadratic function which have a particular significance. The position of these points can be determined using algebra.

(i) **The *y*-intercept**. A quadratic function **always** crosses the *y*-axis. At each point on the *y*-axis, the value of *x* is zero. So the *y*-axis is the line $x = 0$. Because of this, the value of the *y*-intercept can be calculated by setting *x* equal to zero in the quadratic function and is always equal to the value of *c*.

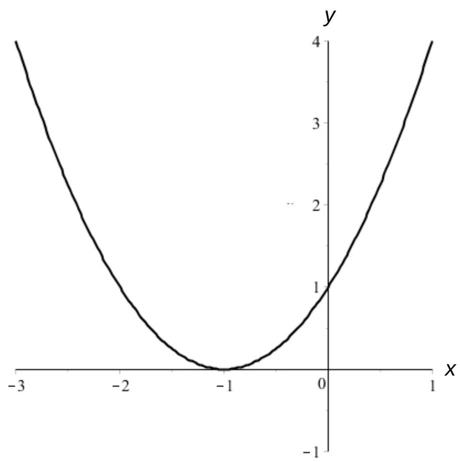
Example: Where does the quadratic function $y = x^2 + 3x + 2$ cross the *y*-axis?

As the function crosses the *y*-axis when $x = 0$, by setting $x = 0$ in the function you find:

$$y = x^2 + 3x + 2 = 0^2 + 3 \cdot 0 + 2 = 2$$

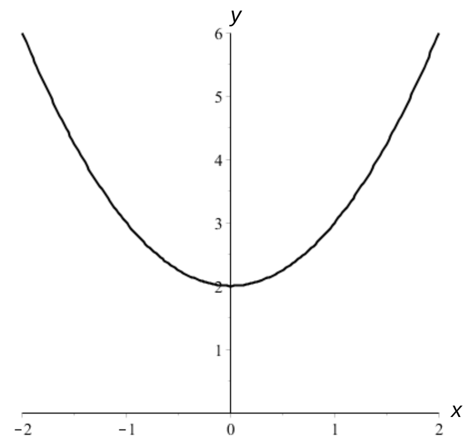
so the function $y = x^2 + 3x + 2$ crosses the *y*-axis at $y = 2$, which is the value of *c*. This is shown explicitly on the graph on the previous page.

(ii) **The *x*-intercept(s)**. The two graphs on the previous page are quadratic functions which cross the *x*-axis at two points. But it is not always true that a quadratic function cuts the *x*-axis. For example $y = x^2 + 2x + 1$ just touches the *x*-axis at a single point, and $y = x^2 + 2$ does not cross the *x*-axis at all.



$y = x^2 + 2x + 1$

Just touches the *x*-axis



$y = x^2 + 2$

Does not cross the *x*-axis

For quadratic functions which cut or touch the *x*-axis, the relevant point(s) can be found by setting $y = 0$ and solving the resulting **quadratic equation**. The solutions of the quadratic equation are known as the **roots**. (If a quadratic function does not cross the *x*-axis then the roots are not real numbers but complex numbers instead.) There are a variety of ways of finding the roots of a quadratic equation, see the study guides: *Solving Quadratic Equations by Factorisation*, *Solving Quadratic Equations Using the Quadratic Formula* and *Completing the Square*).

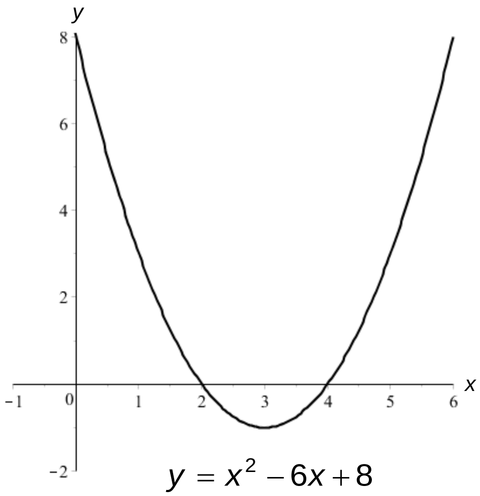
(iii) **The point at which the graph changes direction**. This point is called either the **stationary point** or the **turning point** of the graph. There is a straightforward way of determining the position of the turning point if you know the quadratic function. The *x*-coordinate of the turning point is the value that controls the **axis of symmetry** of the parabola and is given by $-b \div 2a$. The corresponding *y*-coordinate can be found by substituting the value of $x = -b \div 2a$ into the original quadratic function.

- If the parabola touches the *x*-axis at a single point, this point is also the turning point.

- If the parabola cuts the *x*-axis, the *x*-coordinate of the stationary point is halfway between the two cut-points as the parabola is symmetrical.

- If the parabola does not cut the *x*-axis the *x*-coordinate of the turning point is halfway between the two complex roots.

Example: Given that the quadratic function $y = x^2 - 6x + 8$ cuts the *x*-axis at $x = 2$ and $x = 4$, find its turning point.



As $a = 1$ and $b = -6$ the value of the *x*-coordinate of the turning point is given by:

$$-\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = \frac{6}{2} = 3$$

which is also halfway between the cut-points of $x = 2$ and $x = 4$.

The *y*-coordinate is found by substituting $x = 3$ into the function $y = x^2 - 6x + 8$, so:

$$y = x^2 - 6x + 8 = 3^2 - 6 \cdot 3 + 8 = 9 - 18 + 8 = -1$$

Hence the turning point is located at the coordinate $(3, -1)$ as shown above.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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