Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Steps into Algebra

Rearranging Equations

This guide offers an introduction to the essential mathematical skill of rearranging equations. It details a neat flow-chart method to help determine which order to carry out the rearrangement and compares this to another, well-known method. The technique will then be used to solve and transpose equations.

Introduction

One of the most common skills required in algebra and science is the ability to rearrange an equation. In mathematics you may be required to **solve** a problem to find the value of a variable which satisfies given conditions for example:

“Solve $\dfrac{6x-4}{3}=12$.”

This question is asking you to find the value of x which makes the mathematics of the left-hand side of the equals sign equal to 12. In science you are often asked to **transpose** a given equation so that it is written in terms of a given variable for example:

“Write $A=\pi r^2$ in terms of r.”

Here you are being asked to re-write the equation in the form of $r = \ldots$. Both these questions require the skill of **rearranging**.

Mathematical inverses

It is crucial to understand mathematical procedures (called **operations**) and their **inverses** if you are to understand how to rearrange an equation. A mathematical inverse undoes an operation. For example, let’s consider the simple mathematical operation of adding. It should be obvious that to undo adding you must subtract; it is common to think that subtraction is the opposite of addition. However you should avoid thinking of opposites and think of subtraction being the *inverse* of addition. The table below lists some common mathematical operations and their inverses which you should become familiar with.

Operation	**Inverse**
adding	subtracting
subtracting	adding
multiplying	dividing
dividing	multiplying
reciprocal (turn upside down)	reciprocal (turn upside down)
$\times -1$	$\times -1$
squaring	square rooting
square rooting	squaring

[Note that $\times -1$ is its own inverse, this is useful. You may expect it to be $\div -1$ and in fact multiplying by -1 is identical to dividing by -1.]

Of course there are many more (see factsheet: *Mathematical Inverses*) and keeping a list handy with your notes is a good idea.

Flow chart method

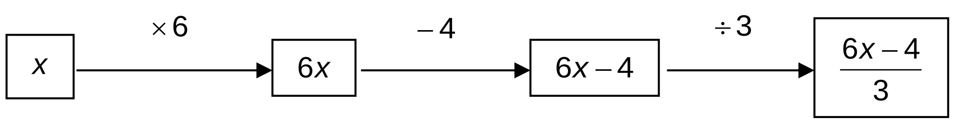
Many students remember being told at school that to rearrange equations, "*you must do the same thing to both sides*". This is certainly true and eventually this is the method that should be employed. However, when learning to rearrange an equation sometimes realising what to actually do to both sides is a problem. If you know how an equation is built then this offers a strategy to undo what has been done and carry out the rearrangement. The **flow chart method** can help you to see how equations are built and hence rearranged. It uses the concept of a **target** which is the variable which you are rearranging for.

Example: As a start let's consider one of the questions above, solve $\frac{6x-4}{3}=12$.

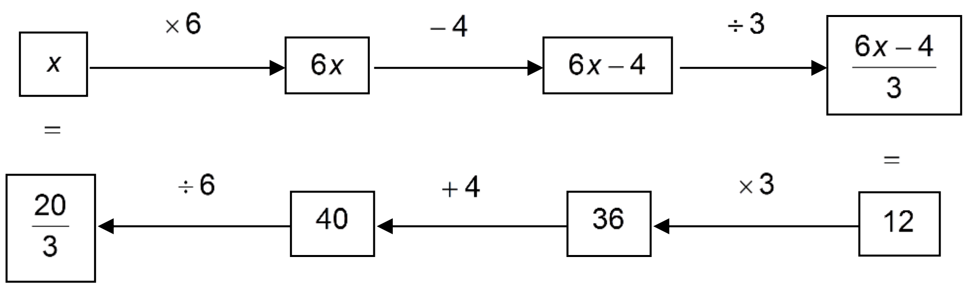
The aim of the flow-chart method is to build the side of the equation where the variable you are considering lies. Here you are looking for the appropriate value of *x* and so *x* is your *target*. You are looking to build the mathematics which contains *x*, in this case:

$$\frac{6x-4}{3}$$

Of course you have to start somewhere and you start with your target *x*. Then, using mathematical operations one at a time you must construct the relevant mathematics. At each stage ask yourself “what do I do next?” and try to make sure that your logical reasoning is correct. Remember you can only use one operation at a time and that you must construct the piece of mathematics exactly. You have three operations in this expression: $\times 6$, -4 and $\div 3$. You must multiply x by 6 first as the 4 is subtracted from $6x$ to make $6x-4$. You must then divide by 3 as the $6x-4$ is all divided by 3. This reasoning is best illustrated as a flow-chart. Begin with your target x in a box. Here the first thing to do is to multiply x by 6 (which is shown as an arrow with the operation above it). Multiplying x by 6 gives $6x$ (which goes in the next box). Then subtract 4 (shown as an arrow with the operation above it) to give $6x-4$ (in the next box). Finally, divide by 3 to give the required expression.



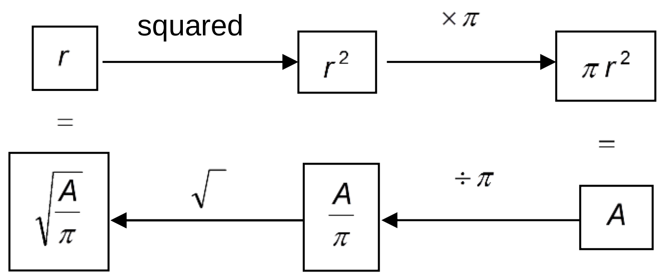
Now for the clever bit, you know that the expression you have built is equal to 12. Underneath your flow-chart, if you start with 12 and go back the other way doing the inverse of the operations indicated above you will solve the problem:



From the starting point of 12, multiply by 3 (because multiplying is the inverse of dividing), then add 4 (as adding is the inverse of subtracting) and finally divide by 6 to find that x is $20/3$. This can be checked by substituting $20/3$ for x in the question and verifying that the result is 12. It is important to line your boxes up correctly, if you do the final box you draw is the solution. **The lower flow-chart is identical to the technique of doing the same thing to both sides.**

Example: Write $A=\pi r^2$ in terms of r.

This time r is your target. Firstly r is squared and then r^2 is multiplied by π. This is equal to A, so returning doing the inverse you must first divide by π and then take the square root.



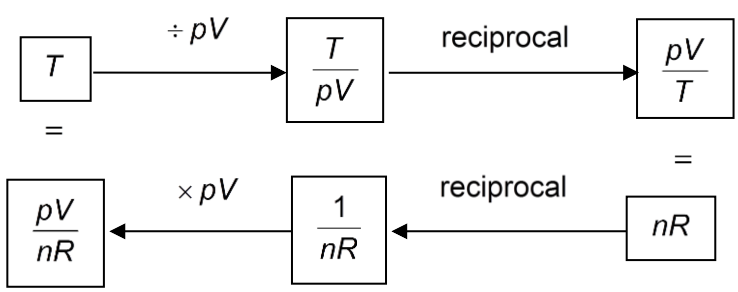
This tells you that $r = \sqrt{\frac{A}{\pi}}$.

Reciprocals

If your target is beneath the dividing line then a useful tactic is to build the relevant side of the equation upside down and then employ the mathematical reciprocal which turns a fraction upside down.

Example: Rearrange $\frac{pV}{T} = nR$ for T.

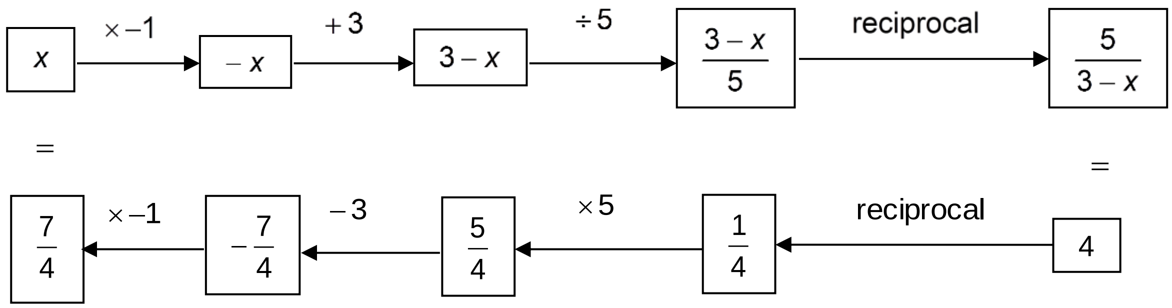
As the target T is beneath the dividing line you should try to build T/pV and then take the reciprocal.



Noticing that nR is the same as the fraction $nR \div 1$ makes the reciprocal easier to perform.

Example: Solve $\frac{5}{3-x} = 4$.

Here the target x is beneath the dividing line and so you need to build the relevant piece of mathematics upside down and then take the reciprocal. This example illustrates the usefulness of multiplying by -1 to change x to $-x$ in the first step. You can think of the minus sign as being created by multiplying by -1. It is also useful to remember that $-x+3=3-x$ for the second step.



Limitations

Although the method works perfectly well for a whole range of rearrangements there are some limitations which you should keep in mind.

1. The method does not work when the target appears more than once in an equation either on one side or on both sides. Take:

$$4t\left(2-\frac{1}{t}\right)=12 \qquad \text{and} \qquad \frac{3}{x-4}=\frac{4}{2-7x}$$

You must use your algebraic skills to manipulate the equations until only one instance of the target is left. If this is not possible then you will not be able to rearrange or solve the equation.

Example: Solve $4t\left(2-\frac{1}{t}\right)=12$.

If you expand the brackets (see study guide: *Opening Brackets*) you find that the left-hand side becomes:

$$4t\left(2-\frac{1}{t}\right)=8t-4$$

and so:

$$8t-4=12$$

which is now suitable for the method (you may use it to find that $t=2$).

2. If the target appears more than once *and* as a different function the procedure

will not work. For example:

$$x^2 + 3x + 2 = 0 \qquad \text{and} \qquad 2x + \sin x = 3$$

In the first case x is both squared and multiplied by 3 and so you have no concrete starting point for the flow chart. Indeed this equation can be solved by factorisation (see study guide: *Solving Quadratic Equations by Factorisation*). However if you **complete the square** first (see study guide: *Completing the Square*) you find that:

$$x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$$

After setting this result to zero you can then use the flow chart method to solve for *x.*

In the second case x is both multiplied by 2 and is the argument of sine, so again you have no fixed starting point. Solution of equations of this type is beyond the scope of these study guides.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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