Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Algebra

Adding and Subtracting Algebraic Fractions

This guide describes how to add and subtract algebraic fractions.

Introduction

An algebraic fraction is a piece of mathematics which includes a dividing line and one or more unknowns or variables. You may think of algebraic fractions as being similar to numerical fractions but the numerator and/or denominator comprise algebraic expressions, instead of just numbers. Some examples are:

$$\frac{x}{2} \qquad \frac{3}{2-x} \qquad \frac{t+2}{x+y^2}$$

It is extremely common in algebra for you to have to manipulate algebraic fractions in some way. This is useful as combining more than one algebraic fraction into a single fraction is beneficial when manipulating equations and expressions. This is usually achieved by the four basic functions of arithmetic: adding, subtracting, multiplying and dividing. The methods for adding, subtracting, multiplying and dividing algebraic fractions are the same as those for numerical fractions. Therefore, if you need to know how to add and subtract algebraic fractions it is important that you understand how to add and subtract numerical fractions. To help you, read the study guide: *Adding and Subtracting Fractions*.

Addition and subtraction of algebraic fractions

When adding and subtracting both numerical and algebraic fractions **you can only add or subtract those with identical denominators**. If the denominators of your fractions are different, you **cannot** add or subtract the fractions. You need to use your knowledge of **equivalent fractions** to re-write the fractions so that they have the same denominator; usually called the **common denominator**.

Finding common denominators

A common denominator is a piece of mathematics which has all the denominators of the fractions you are trying to add or subtract as **factors.** Methods for finding common denominators for numerical fractions are discussed in the study guide: *Adding and Subtracting Fractions*. Calculating a common denominator for algebraic fractions requires you to identify which type of denominators you have.

(i) **Denominators containing only numbers**. Such as:

$$\frac{a}{4}+\frac{4a}{3}$$

Here you follow the same procedure as that for numerical fractions. Specifically find the **lowest common multiple** of the numbers, see study guide: *Lowest Common Multiple*.

(ii) **Algebraic denominators which do not share a common factor**. Such as:

$$\frac{1}{x}-\frac{2}{y}$$

Here the common denominator is found by simply multiplying together the denominators. In the fractions above it is clear that no factors are shared because x is different from y. This will not always be the case. You should always factorise your denominators in order to help you to see if they share any factors. Factorisation also helps you when you have to write equivalent fractions. If you have difficulty factorising expressions you should read the study guides: *Simple Factorisation* and *Factorising Quadratic Expressions*.

(iii) **Algebraic denominators with common factors.** Such as:

$$\frac{2}{xy^3}-\frac{1}{xy}$$

Remember, your common denominator should contain *all* the factors of *all* the denominators. In cases where some factors are shared you may only have adjust certain denominators to ensure this rule is met. The denominators of the two fractions above share the factor xy. To make the denominators the same you only have to multiply xy by y^2. Simply multiplying the denominators together gives a perfectly reasonable, but over-complicated, common denominator of x^2y^4. This, in turn, leads to more complicated equivalent fractions and more intricate algebraic manipulation.

Some examples

Example: What is $\frac{a}{4}+\frac{4a}{3}$ expressed as a single fraction?

This is an example of the first type of denominator: denominators containing only numbers. The lowest common denominator in this case is 12. To find the equivalent fractions, the numerator and denominator of the first fraction are multiplied by 3 and the numerator and denominator of the second fraction are multiplied by 4. Once this is done the fractions can be added to find that:

$$\frac{a}{4}+\frac{4a}{3}=\frac{3a}{12}+\frac{16a}{12}=\frac{3a+16a}{12}=\frac{19a}{12}$$

Example: What is $\frac{2}{x}+\frac{1}{1-x}$ expressed as a single fraction?

This is an example of the second type of denominator: algebraic denominators which do not share a common factor. The common denominator is found by multiplying the two denominators to give $x(1-x)$. To find the equivalent fractions, the numerator and denominator of the first fraction are multiplied by $(1-x)$ and the numerator and denominator of the second fraction are multiplied by *x*. You can now perform the addition and simplify the numerator as follows:

$$\frac{2}{x}+\frac{1}{1-x}=\frac{2(1-x)}{x(1-x)}+\frac{x}{x(1-x)}=\frac{2-2x+x}{x(1-x)}=\frac{2-x}{x(1-x)}$$

Example: What is $\frac{3c}{a^2b}-\frac{1}{ab^2}$ written as a single fraction?

This is an example of the third type of denominator: algebraic denominators with common factors. In this example the denominators have a common factor of ab. The first denominator is ab multiplied by *a* and the second denominator is ab multiplied by *b*. All the factors contained by the two denominators are ab, *a*, and *b.* When these are multiplied together you get the simplest common denominator, a^2b^2.

To find the equivalent fractions, the numerator and denominator of the first fraction are multiplied by *b* and the numerator and denominator of the second fraction are multiplied by *a*. You can now perform the subtraction as follows:

$$\frac{3c}{a^2b}-\frac{1}{ab^2}=\frac{3bc}{a^2b^2}-\frac{a}{a^2b^2}=\frac{3bc-a}{a^2b^2}$$

Example: What is $\dfrac{2}{x+7}+\dfrac{1}{x^2+9x+14}$ written as a single fraction?

The first stage here is to factorise the quadratic expression $x^2+9x+14$ to give $(x+2)(x+7)$. So the problem becomes:

$$\frac{2}{x+7}+\frac{1}{(x+2)(x+7)}$$

There is a common factor in the denominators of $(x+7)$. This is already the denominator of the first fraction and is multiplied by $(x+2)$ in the denominator of the second fraction, so you have the factors $(x+2)$ and $(x+7)$. Multiplying these together gives the common denominator $(x+2)(x+7)$. To find the equivalent fractions, the numerator and denominator of the first fraction are multiplied by $(x+2)$, the second fraction already has the denominator $(x+2)(x+7)$ and so you leave it alone. You can now perform the addition as follows:

$$\frac{2}{x+7}+\frac{1}{(x+2)(x+7)}=\frac{2(x+2)}{(x+2)(x+7)}+\frac{1}{(x+2)(x+7)}=\frac{2x+4+1}{(x+2)(x+7)}=\frac{2x+5}{(x+2)(x+7)}$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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Ask:	ask.let@uea.ac.uk
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