Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Adding and Subtracting Algebraic Fractions

Adding and Subtracting Algebraic Fractions study guide

1. In these questions the denominators are the same in (common to) both fractions. You can add or subtract the numerators directly and put the result over the common denominator to find the answer.

(a) $\frac{1}{x}+\frac{1}{x}=\frac{1+1}{x}=\frac{2}{x}$

The common denominator is x. The numerators are 1 and 1 which add to give the new numerator 2.

Putting the new numerator over the denominator gives $\frac{2}{x}$.

(b) $\frac{1}{x}-\frac{1}{x}=\frac{1-1}{x}=\frac{0}{x}=0$

The common denominator is x. The numerators are 1 and 1 which subtract to give the new numerator 0.

Putting the new numerator over the denominator gives $\frac{0}{x}=0$.

Alternatively you might see that anything subtracted from itself is 0.

(c) $\frac{1}{x}+\frac{y}{x}=\frac{1+y}{x}$

The denominator is x. The numerators are 1 and y which add to give the new numerator $1+y$.

Putting the new numerator over the denominator gives $\frac{1+y}{x}$.

(d) $\frac{1}{3x}+\frac{4}{3x}=\frac{1+4}{3x}=\frac{5}{3x}$

The common denominator is $3x$. The numerators are 1 and 4 which add to give the new numerator 5.

Putting the new numerator over the denominator gives $\frac{5}{3x}$.

(e) $\frac{5}{2a}-\frac{1}{2a}=\frac{5-1}{2a}=\frac{4}{2a}=\frac{2\times 2}{2\times a}=\frac{2}{a}$

The common denominator is $2a$. The numerators are 5 and 1 which subtract to give the new numerator 4.

Putting the new numerator over the denominator gives $\frac{4}{2a}$.

The numerator have a common factor of 2 which can be cancelled down to give $\frac{2}{a}$.

(f) $\frac{1}{1+x}+\frac{7}{1+x}=\frac{1+7}{1+x}=\frac{8}{1+x}$

The common denominator is $1+x$. The numerators are 1 and 7 which add to give the new numerator 8.

Putting the new numerator over the denominator gives $\frac{8}{1+x}$.

(g) $\frac{3x}{p+q}-\frac{2x}{p+q}=\frac{3x-2x}{p+q}=\frac{x}{p+q}$

The common denominator is $p+q$. The numerators are $3x$ and $2x$ which subtract to give the new numerator x.

Putting the new numerator over the denominator gives $\frac{x}{p+q}$.

(h) $\frac{s+t}{2u-v}+\frac{s-t}{2u-v}=\frac{s+t+s-t}{2u-v}=\frac{2s}{2u-v}$

The common denominator is $2u-v$. The numerators are $s+t$ and $s-t$ which add to give the new numerator $2s$.

Putting the new numerator over the denominator gives $\frac{2s}{2u-v}$.

(i) $-\frac{1}{c^2}+\frac{1}{c^2}=\frac{1}{c^2}-\frac{1}{c^2}=\frac{1-1}{c^2}=\frac{0}{c^2}=0$

It is perfectly fine to write this question as $\frac{1}{c^2}-\frac{1}{c^2}$.

The common denominator is c^2. The numerators are 1 and 1 which subtract to give the new numerator 0.

Putting the new numerator over the denominator gives $\frac{0}{c^2}=0$.

Alternatively you might see that anything subtracted from itself is 0.

2.

(a) $\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}$

The common denominator is found by multiplying the denominator of the first fraction which is *x,* by the denominator of the second fraction which is *y*, to give xy.

Then you can rewrite the fractions as equivalent fractions with the new denominator.

For the first fraction to have a denominator of xy you must multiply the numerator and denominator by y to give y/xy which is allowed because this is equivalent to $1/x$.

For the second fraction to have a denominator of xy you must multiply the numerator and denominator by x to give x/xy which is allowed because this is equivalent to $1/y$.

Then the fractions have the same denominator and so you can add the numerators and put them over the new denominator to give the answer.

(b) $\frac{1}{x}-\frac{1}{y}=\frac{y}{xy}-\frac{x}{xy}=\frac{x-y}{xy}$

The common denominator is found by multiplying the denominator of the first fraction which is *x*, by the denominator of the second fraction which is *y*, to give xy. Then you can rewrite the fractions as equivalent fractions with the new denominator.

For the first fraction to have a denominator of xy you must multiply the numerator and denominator by y to give y/xy which is allowed because this is equivalent to $1/x$.

For the second fraction to have a denominator of xy you must multiply the numerator and denominator by x to give x/xy which is allowed because this is equivalent to $1/y$.

Then the fractions have the same denominator and so you can subtract the numerators and put them over the new denominator to give the answer.

(c) $$\frac{a}{b}+\frac{c}{d}=\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$$

The common denominator is found by multiplying the denominator of the first fraction which is b, by the denominator of the second fraction which is d, to give bd. Then you can rewrite the fractions as equivalent fractions with the new denominator.

For the first fraction to have a denominator of bd you must multiply the numerator and denominator by d to give ad/bd which is allowed because this is equivalent to a/b.

For the second fraction to have a denominator of bd you must multiply the numerator and denominator by b to give bc/bd which is allowed because this is equivalent to c/d.

Then the fractions have the same denominator and so you can add the numerators and put them over the new denominator to give the answer.

(d) $$\frac{4}{x}+\frac{3}{4y}=\frac{16y}{4xy}+\frac{3x}{4xy}=\frac{16y+3x}{4xy}$$

The common denominator is found by multiplying the denominator of the first fraction which is x, by the denominator of the second fraction which is $4y$, to give $4xy$.

Then you can rewrite the fractions as the equivalent fractions $\frac{16y}{4xy}$ and $\frac{3x}{4xy}$.

These fractions have the same denominator and so you can add to the numerators and put them over the new denominator to give the answer.

(e) $$\frac{7}{x+1}-\frac{5}{y+1}=\frac{7(y+1)}{(x+1)(y+1)}-\frac{5(x+1)}{(x+1)(y+1)}$$
$$=\frac{7(y+1)-5(x+1)}{(x+1)(y+1)}=\frac{7y+7-5x-5}{(x+1)(y+1)}=\frac{7y-5x+2}{(x+1)(y+1)}$$

The common denominator is found by multiplying the denominator of the first fraction which is $x+1$, by the denominator of the second fraction which is $y+1$. It is helpful to use brackets because you need the whole of $x+1$ multiplying the whole of $y+1$ so the common denominator is $(x+1)(y+1)$.

Then you can rewrite the fractions as the equivalent fractions $\frac{7(y+1)}{(x+1)(y+1)}$ and $\frac{5(x+1)}{(x+1)(y+1)}$.

These fractions have the same denominator and so you can subtract to the numerators and put them over the new denominator. This answer is ok at this stage but in order to keep the numerator as simple as possible you need to open the brackets and collect any like terms. See the study guide: *Opening Brackets* for help with this step.

(f)
$$\frac{p}{p-q}-\frac{a}{a+b}=\frac{p(a+b)}{(p-q)(a+b)}-\frac{a(p-q)}{(p-q)(a+b)}$$
$$=\frac{p(a+b)-a(p-q)}{(p-q)(a+b)}=\frac{ap+bp-ap+aq}{(p-q)(a+b)}=\frac{bp+aq}{(p-q)(a+b)}$$

The common denominator is found by multiplying the denominator of the first fraction which is $p-q$, by the denominator of the second fraction which is $a+b$ so the common denominator is $(p-q)(a+b)$.

Then you can rewrite the fractions as $\frac{p(a+b)}{(p-q)(a+b)}$ and $\frac{a(p-q)}{(p-q)(a+b)}$.

These fractions have the same denominator and so you can subtract to the numerators and put them over the new denominator. This answer is ok at this stage but in order to keep the numerator as simple as possible you need to open the brackets and collect any like terms. See the study guide: *Opening Brackets* for help with this step.

(g)
$$\frac{v}{2u+v}-\frac{1}{s}=\frac{vs}{(2u+v)s}-\frac{(2u+v)}{(2u+v)s}=\frac{vs-(2u+v)}{(2u+v)s}$$

The common denominator is found by multiplying the denominator of the first fraction which is $2u+v$, by the denominator of the second fraction which is s, to give $(2u+v)s$.

Then you can rewrite the fractions as the equivalent fractions $\frac{vs}{(2u+v)s}$ and $\frac{2u+v}{(2u+v)s}$

These fractions have the same denominator and so you can subtract the numerators and put them over the new denominator to give the answer.

(h) $$\frac{uvw}{abc}+\frac{xyz}{def}=\frac{defuvw}{abcdef}+\frac{abcxyz}{abcdef}=\frac{defuvw+abcxyz}{abcdef}$$

The common denominator is found by multiplying the denominator of the first fraction which is abc, by the denominator of the second fraction which is def , to give $abcdef$.

Then you can rewrite the fractions as the equivalent fractions $\frac{defuvw}{abcdef}$ and $\frac{abcxyz}{abcdef}$.

These fractions have the same denominator and so you can subtract the numerators and put them over the new denominator to give the answer.

(i) $$\frac{1}{x^3}-\frac{2}{y^2}=\frac{y^2}{x^3y^2}-\frac{2x^3}{x^3y^2}=\frac{y^2-2x^3}{x^3y^2}$$

The common denominator is found by multiplying the denominator of the first fraction which is x^3, by the denominator of the second fraction which is y^2, to give x^3y^2.

Then you can rewrite the fractions as the equivalent fractions $\frac{y^2}{x^3y^2}$ and $\frac{2x^3}{x^3y^2}$.

These fractions have the same denominator and so you can subtract the numerators and put them over the new denominator to give the answer.

3.

(a) $2x$

There is a factor of 2 in both of these expressions. There is also a factor of x in the first expression and therefore the simplest common multiple is found by multiplying this with the common factor to give $2x$. Then $2x$ has both 2 and $2x$ as factors. Note that $4x$ is a common multiple but not the simplest one.

(b) $2x$

There is a factor of x in both of these expressions. There is also a factor of 2 in the first expression and therefore the simplest common multiple is found by multiplying this with the common factor to give $2x$. Then $2x$ has both $2x$ and x as factors. Note that $2x^2$ is a common multiple but not the simplest one.

(c) x^2

There is a factor of x in both of these expressions. There is another factor of x in the first expression and therefore the simplest common multiple is found by multiplying this with the common factor to give x^2. Then x^2 has both x^2 and x as factors. Note that x^3 is a common multiple but not the simplest one.

(d) 9

The number 9 has both 9 and 3 as factors. Note that 18 is a common multiple but not the simplest one.

(e) abc

The expression abc has abc and ab and c as factors. Note that a^3b^2c is a common multiple but not the simplest one.

(f) $a^4b^2c^5$

The expression $a^4b^2c^5$ has both a^2bc^5 and $a^4b^2c^3$ as factors. Note that $a^6b^3c^8$ is a common multiple but not the simplest one.

4.

(a) $$\frac{1}{2x}+\frac{1}{2}=\frac{1}{2x}+\frac{x}{2x}==\frac{1+x}{2x}$$

Use the denominator $2x$ from question 3a.

(b) $$\frac{1}{2x}+\frac{1}{x}=\frac{1}{2x}+\frac{2}{2x}=\frac{1+2}{2x}=\frac{3}{2x}$$

Use the denominator $2x$ from question 3b.

(c) $\frac{1}{x^2}+\frac{1}{x}=\frac{1}{x^2}+\frac{x}{x^2}=\frac{1+x}{x^2}$

Using the denominator x^2 from question 3c.

(d) $\frac{7y}{9}-\frac{x}{3}=\frac{7y}{9}-\frac{3x}{9}=\frac{7y-3x}{9}$

Using the denominator of 9 from question 3d.

(e) $\frac{2d}{abc}-\frac{3f}{ab}+\frac{1}{a}=\frac{2d}{abc}-\frac{3cf}{abc}+\frac{bc}{abc}=\frac{2d-3cf+bc}{abc}$

Using the denominator of abc from question 3e.

(f) $\frac{5}{a^2bc^5}-\frac{1}{a^4b^2c^3}=\frac{5a^2b}{a^4b^2c^5}-\frac{c^2}{a^4b^2c^5}=\frac{5a^2b-c^2}{a^4b^2c^5}$

Using the denominator of $a^4b^2c^5$ from question 3f.

5.

(a) $\frac{2+x}{2x}$

Multiplying the denominators gives $2x$. Using this as the new denominator and writing the equivalent fractions gives you $\frac{x}{2x}+\frac{2}{2x}=\frac{2+x}{2x}$ which cannot be cancelled down.

(b) $\frac{1-x}{2(x+1)}$

Multiplying the denominators gives $2(x+1)$. Using this as the new denominator and writing the equivalent fractions gives you $\frac{2}{2(x+1)}-\frac{x+1}{2(x+1)}=\frac{2-(x+1)}{2(x+1)}=\frac{1-x}{2(x+1)}$ which cannot be cancelled down.

(c) $\frac{3x-1}{2x}$

Using the simplest common denominator $2x$ gives $\frac{1+x}{2x}+\frac{2x-2}{2x}=\frac{3x-1}{2x}$ which cannot be cancelled down.

(d) $\dfrac{x+5}{(x+2)(x+1)}$

Multiplying the denominators gives $(x+2)(x+1)$. Using this as the new denominator and writing the equivalent fractions gives you:

$$\frac{(x-1)(x+1)}{(x+1)(x+2)} - \frac{(x-3)(x+2)}{(x+1)(x+2)} = \frac{(x-1)(x+1)-(x-3)(x+2)}{(x+1)(x+2)}$$

expanding the brackets gives the answer.

(e) $\dfrac{ab^2 - ad^2}{bc^2d^3}$

Using the simplest common denominator bc^2d^3 gives $\dfrac{ab^2}{bc^2d^3} - \dfrac{ad^2}{bc^2d^3} = \dfrac{ab^2 - ad^2}{bc^2d^3}$ which cannot be cancelled down.

(f) $\dfrac{ab^2c - acd^2 + b^2d}{bc^2d^3}$

You can use the answer from part (e) to add the first two fractions.

Then, using the same denominator gives $\dfrac{ab^2 - ad^2}{bc^2d^3} + \dfrac{b^2d}{bc^2d^3} = \dfrac{ab^2 - ad^2 + b^2d}{bc^2d^3}$ which cannot be cancelled down.

(g) $\dfrac{x^2 + y^2}{xy}$

Multiplying the denominators gives xy. Using this as the new denominator and writing the equivalent fractions gives you $\dfrac{x^2}{xy} + \dfrac{y^2}{xy} = \dfrac{x^2 + y^2}{xy}$ which cannot be cancelled down.

6.

(a) $\dfrac{x+1}{x}$

By writing 1 as $\dfrac{1}{1}$ and using x as the common denominator you get:

$$\frac{1}{1} + \frac{1}{x} = \frac{x}{x} + \frac{1}{x} = \frac{x+1}{x}.$$

(b) $\frac{2x+1}{2}$

By writing x as $\frac{x}{1}$ and using 2 as the common denominator you get:

$$\frac{x}{1}+\frac{1}{2}=\frac{2x}{2}+\frac{1}{2}=\frac{2x+1}{2}$$

(c) $\frac{25x^2+1}{5x}$

By writing $5x$ as $\frac{5x}{1}$ and using $5x$ as the common denominator you get:

$$\frac{5x}{1}+\frac{1}{5x}=\frac{25x^2}{5x}+\frac{1}{5x}=\frac{5x^2+1}{5x}$$

(d) $\frac{x^2+x+1}{x}$

By writing x as $\frac{x}{1}$ and 1 as $\frac{1}{1}$ and also using x as the common denominator you get:

$$\frac{x}{1}+\frac{1}{1}+\frac{1}{x}=\frac{x^2}{x}+\frac{x}{x}+\frac{1}{x}=\frac{x^2+x+1}{x}$$

7.

You can perform the additions (a), (b) and (f) to find that:

(a) $\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{x+y}{xy}$

(b) $1+\frac{y}{x}=\frac{x}{x}+\frac{y}{x}=\frac{x+y}{x}$

(f) $2+\frac{x-y}{y}=\frac{2y}{y}+\frac{x-y}{y}=\frac{2y+x-y}{y}=\frac{x+y}{y}$

Using these results you can see the (a) and (e) are equivalent, (b) and (c) are equivalent and (d) and (f) are equivalent.

8.

(a) $\frac{1}{x}+\frac{1}{x^2}=\frac{x}{x^2}+\frac{1}{x^2}=\frac{x+1}{x^2}$

Use the simplest common denominator of x^2.

(b) $$\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}=\frac{x^2}{x^3}+\frac{x}{x^3}+\frac{1}{x^3}=\frac{x^2+x+1}{x^2}$$

Use the simplest common denominator of x^3.

(c) $$\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\ldots+\frac{1}{x^n}=\frac{x^{n-1}}{x^n}+\frac{x^{n-2}}{x^n}+\frac{x^{n-3}}{x^3}+\ldots+\frac{1}{x^n}=\frac{x^{n-1}+x^{n-2}+\ldots+x^2+x+1}{x^n}$$

Use the simplest common denominator of x^n.