Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Steps into Algebra

Multiplying and Dividing Algebraic Fractions

This guide describes how to multiply and divide algebraic fractions.

Introduction

An algebraic fraction is a piece of mathematics which includes a dividing line and one or more unknowns or variables. You may think of algebraic fractions as being similar to numerical fractions but the numerator and/or denominator comprise algebraic expressions, instead of just numbers. Some examples are:

$$\frac{x}{2} \qquad \frac{3}{2-x} \qquad \frac{t+2}{x+y^2}$$

It is extremely common in algebra for you to have to manipulate algebraic fractions in some way. This is useful as combining more than one algebraic fraction into a single fraction is beneficial when manipulating equations and expressions. This is usually achieved by the four basic functions of arithmetic: adding, subtracting, multiplying and dividing. The methods for adding, subtracting, multiplying and dividing algebraic fractions are the same as those for numerical fractions. Therefore, if you need to know how to multiply and divide algebraic fractions it is important that you understand how to multiply and divide numerical fractions. To help you, read the study guide: *Multiplying and Dividing Fractions*.

Multiplying algebraic fractions

When multiplying numerical fractions, you multiply the numerators to find the numerator of the answer and multiply the denominators to find the denominator of the answer. You should then cancel down the result if possible; see the study guide: *Cancelling Down Fractions* if you find this difficult. For example:

$$\frac{3}{8}\times\frac{2}{5}=\frac{3\times 2}{8\times 5}=\frac{6}{40}=\frac{3}{20} \quad .$$

The method for multiplying algebraic fractions is identical, that is you **multiply the numerators of the fractions to find the numerator of the answer and multiply the denominators of the fractions to find the denominator of the answer**. You can use the SNALPHABET system to help you perform the multiplications, see study guide: *SNALPHABET*. When you have done this you should **cancel down** your result if necessary. To cancel down the algebraic fraction you need to factorise both the numerator and denominator and write them in factored form, see study guides: *Simple Factorisation* and *Factorising Quadratic Expressions*. When you have the numerator and denominator in factored form you cancel down identical factors to give the simplified form of the algebraic fraction.

Example: What is $\frac{s}{2} \times \frac{3t}{s^2}$?

Multiplying the numerators together gives $3st$ and multiplying the denominators together gives $2s^2$, so:

$$\frac{s}{2} \times \frac{3t}{s^2} = \frac{3st}{2s^2}$$

Writing in factored form and cancelling the common factor of s gives the simplified form of the algebraic fraction:

$$\frac{3st}{2s^2} = \frac{3 \times \cancel{s} \times t}{2 \times \cancel{s} \times s} = \frac{3t}{2s}$$

Example: What is $\frac{3}{4q-8} \times \frac{q-2}{q+1}$?

When you have numerators or denominators containing addition or subtraction (such as $q+1$ and $4q-8$ in the example) it is helpful to put them in brackets before multiplying. So the question can be re-written as:

$$\frac{3}{(4q-8)} \times \frac{(q-2)}{(q+1)} \quad .$$

Multiplying the numerators gives $3(q-2)$ and multiplying the denominators gives $(4q-8)(q+1)$. The terms in the first bracket in the denominator have a common factor of 4 and so can be factorised to give $4(q-2)$. So, after performing the multiplication, you get:

$$\frac{3}{(4q-8)} \times \frac{(q-2)}{(q+1)} = \frac{3(q-2)}{4(q-2)(q+1)} \quad .$$

There is a common factor of $(q-2)$ which can be cancelled down to leave the simplified answer:

$$\frac{3\cancel{(q-2)}}{4\cancel{(q-2)}(q+1)} = \frac{3}{4(q+1)} \quad .$$

Example: Simplify $\frac{x+5}{x+3} \times \frac{x^2+4x+3}{x^2+9x+20}$.

In this example quadratic expressions make up the numerator and denominator of the second fraction. The numerator x^2+4x+3 can be factorised to give $(x+1)(x+3)$ and the denominator $x^2+9x+20$ can be factorised to give $(x+5)(x+4)$. So, after introducing brackets and factorising the quadratic expressions:

$$\frac{x+5}{x+3} \times \frac{x^2+4x+3}{x^2+9x+20} = \frac{(x+5)}{(x+3)} \times \frac{(x+1)(x+3)}{(x+5)(x+4)} = \frac{(x+5)(x+1)(x+3)}{(x+3)(x+5)(x+4)}$$

There are common factors of $x+3$ and $x+5$ which can be cancelled down to give:

$$\frac{\cancel{(x+5)}(x+1)\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x+5)}(x+4)} = \frac{x+1}{x+4} \quad .$$

Division of algebraic fractions

As with multiplying algebraic fractions, the method for dividing algebraic fractions is identical to that for dividing numerical fractions. Specifically you take **the reciprocal of the second fraction** (in other words, turn it upside down) **and then multiply the algebraic fractions together instead of dividing them**. You should then simplify your answer by cancelling down if possible.

Example: Write $\frac{xy}{3} \div \frac{y^2}{9x}$ in its simplest form.

Inverting the second fraction and multiplying instead gives:

$$\frac{xy}{3} \div \frac{y^2}{9x} = \frac{xy}{3} \times \frac{9x}{y^2}$$

After performing the multiplication, the numerator and denominator can be written in factored form. You have a common factor of $3y$ which can be cancelled down to give:

$$\frac{xy}{3}\times\frac{9x}{y^2}=\frac{9x^2y}{3y^2}=\frac{\cancel{3}\cdot 3\cdot x\cdot x\cdot \cancel{y}}{\cancel{3}\cdot \cancel{y}\cdot y}=\frac{3x^2}{y}$$

Example: What is $\dfrac{2a}{a+3b}\div\dfrac{4a^2}{b}$?

Here it is useful to begin by putting the denominator of the first fraction in brackets, now you invert the second fraction and multiply to give:

$$\frac{2a}{(a+3b)}\div\frac{4a^2}{b}=\frac{2a}{(a+3b)}\times\frac{b}{4a^2}=\frac{2ab}{4a^2(a+3b)}$$

After writing the algebraic fraction in factored form you can see that there is a common factor of $2a$. This can be cancelled down to give the simplified form:

$$\frac{2ab}{4a^2(a+3b)}=\frac{\cancel{2}\cdot \cancel{a}\cdot b}{\cancel{2}\cdot 2\cdot \cancel{a}\cdot a\cdot (a+3b)}=\frac{b}{2a(a+3b)}$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761
Ask: ask.let@uea.ac.uk
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