Learning Enhancement Team

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Model Answers: Multiplying and Dividing Algebraic Fractions

Multiplying and Dividing Algebraic Fractions study guide

1) You use the same method to multiply algebraic fractions as you would to multiply numerical fractions. To find the numerator of the answer you multiply the numerators of the fractions in the question. Similarly to find the denominator of the answer you multiply the denominators of the fractions in the question. You can use the SNALPHABET system to help you here (see study guide: *SNALPHABET*).

After this you should **simplify** your answer, if you can, by identifying and **cancelling down** common factors in your numerator and denominator. If you have difficulty finding common factors, the study guide: *Simple Factorisation* can help.

a) $\frac{3}{x} \times \frac{y}{7} = \frac{3y}{7x}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

b) $\frac{6y}{x^2} \times \frac{z}{7} = \frac{6yz}{7x^2}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

c) $\frac{1}{k^2} \times \frac{v}{10} = \frac{v}{10k^2}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

d) In this question it is useful to use brackets to keep the $4+s$ denominator together and help to stop cancelling errors.

$$\frac{4p^3}{4+s}\times\frac{s}{t}=\frac{4p^3}{(4+s)}\times\frac{s}{t}=\frac{4p^3}{(4+s)t}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors. You may be tempted to cancel down the 4 but in the numerator it is inside the brackets. Remember cancelling down is about dividing and you cannot separate the 4 from the addition of *s*.

2)

a) $\frac{2}{1}\times\frac{y}{9}=\frac{2y}{9}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

b) It is common to multiply both numerator and denominator by 2 in questions like this. However this would be incorrect. You need to express the number 2 as a fraction (2/1) and multiply the fractions together in the usual way. You should realise that this question is identical to the previous question.

$$2\times\frac{y}{9}=\frac{2}{1}\times\frac{y}{9}=\frac{2y}{9}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

c) $4y\times\frac{y}{7x^2}=\frac{4y}{1}\times\frac{y}{7x^2}=\frac{4y^2}{7x^2}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

d) $\frac{\lambda}{8}\times 3\mu=\frac{\lambda}{8}\times\frac{3\mu}{1}=\frac{3\lambda\mu}{8}$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

3)

a) $\frac{3}{s} \times \frac{s}{7} = \frac{3s}{7s}$

There is a common factor of s in the numerator and denominator which can be cancelled down to give:

$$\frac{3s}{7s} = \frac{3 \times \cancel{s}}{7 \times \cancel{s}} = \frac{3}{7}$$

b) $\frac{r^2}{s^2} \times \frac{s}{r} = \frac{r^2 s}{rs^2}$

There is a common factor of rs in the numerator and denominator which can be cancelled down to give:

$$\frac{r^2 s}{rs^2} = \frac{r \times \cancel{rs}}{s \times \cancel{rs}} = \frac{r}{s}$$

c) $\frac{3}{x^2} \times \frac{x}{7} = \frac{3x}{7x^2}$

There is a common factor of x in the numerator and denominator which can be cancelled down to give:

$$\frac{3x}{7x^2} = \frac{3 \times \cancel{x}}{7x \times \cancel{x}} = \frac{3}{7x}$$

d) $\frac{12p^3}{q^2} \times \frac{q^5}{4p^4} = \frac{12p^3q^5}{4p^4q^2}$

There is a common factor of $4p^3q^2$ in the numerator and denominator which can be cancelled down to give:

$$\frac{12p^3q^5}{4p^4q^2} = \frac{3q^3 \times \cancel{4p^3q^2}}{p \times \cancel{4p^3q^2}} = \frac{3q^3}{p}$$

e) $$\frac{81m^6}{n^4}\times\frac{4n^7m}{9}=\frac{324m^7n^7}{9n^4}$$

There is a common factor of $9n^4$ in the numerator and denominator which can be cancelled down to give:

$$\frac{324m^7n^7}{9n^4}=\frac{36m^7n^3\times\cancel{9n^4}}{1\times\cancel{9n^4}}=\frac{36m^7n^3}{1}=36m^7n^3$$

f) $$\frac{6x^3y^5z^2}{10x^4y^3z^4}\times\frac{5x^2y^2z^6}{3xy^4z^4}=\frac{30x^5y^7z^8}{30x^5y^7z^8}$$

Here the numerator and denominator are identical and so the answer is 1.

g) $$\frac{4f^3}{(u+3)^2}\times\frac{u+3}{t}=\frac{4f^3(u+3)}{(u+3)^2t}$$

There is a common factor of $(u+3)$ in the numerator and denominator which can be cancelled down to give:

$$\frac{4f^3(u+3)}{(u+3)^2t}=\frac{4f^3\times\cancel{(u+3)}}{(u+3)t\times\cancel{(u+3)}}=\frac{4f^3}{(u+3)t}$$

h) $$\frac{-7\pi r^3}{2\theta}\times\frac{K_0\theta}{14\pi r^2}=-\frac{7\pi K_0\theta r^3}{28\pi\theta r^2}$$

There is a common factor of $7\pi\theta r^2$ in the numerator and denominator which can be cancelled down to give:

$$-\frac{7\pi K_0\theta r^3}{28\pi\theta r^2}=-\frac{K_0r\times\cancel{7\pi\theta r^2}}{4\times\cancel{7\pi\theta r^2}}=-\frac{K_0r}{4}$$

i) $$\frac{4p^3}{(4+s)^2}\times-\frac{4+s}{t}=-\frac{4(4+s)p^3}{(4+s)^2t}$$

There is a common factor of $(4+s)$ in the numerator and denominator which can be cancelled down to give:

$$-\frac{4(4+s)p^3}{(4+s)^2t}=-\frac{4p^3\times\cancel{(4+s)}}{(4+s)\cancel{t}\times\cancel{(4+s)}}=-\frac{4p^3}{(4+s)t}$$

j) $$(x-1)\times\frac{-1}{x+1}=\frac{(x-1)}{1}\times\frac{-1}{x+1}=\frac{-1\times(x-1)}{(x+1)}$$

You can open the brackets in the numerator to explore whether you have common factors to cancel down:

$$\frac{-1\times(x-1)}{(x+1)}=\frac{-x+1}{(x+1)}=\frac{1-x}{(x+1)}=\frac{(1-x)}{(x+1)}$$

So there are no common factors to cancel down.

k) $$(-x-1)\times\frac{1}{x+1}=\frac{(-x-1)}{1}\times\frac{1}{(x+1)}=\frac{(-x-1)}{(x+1)}$$

There looks like there are no common factors to cancel down but you can factorise the numerator and reveal a factor which is common to the numerator and denominator:

$$\frac{(-x-1)}{(x+1)}=\frac{-1(x+1)}{(x+1)}=\frac{-1\times\cancel{(x+1)}}{1\times\cancel{(x+1)}}=\frac{-1}{1}=-1$$

l) $$\frac{1}{v+3}\times(3v+9)=\frac{1}{(v+3)}\times\frac{(3v+9)}{1}=\frac{(3v+9)}{(v+3)}$$

There looks like there are no common factors to cancel down but you can factorise the numerator and reveal a factor which is common to the numerator and denominator:

$$\frac{(3v+9)}{(v+3)}=\frac{3(v+3)}{(v+3)}=\frac{3\times\cancel{(v+3)}}{1\times\cancel{(v+3)}}=\frac{3}{1}=3$$

m) $$5\times\frac{3/5}{g-3}$$

This is quite a complicated question and so you may want to break it down into smaller parts. Firstly perform the multiplication and use brackets to help keep things together:

$$5\times\frac{3/5}{g-3}=\frac{5}{1}\times\frac{(3/5)}{(g-3)}=\frac{5\times(3/5)}{(g-3)}$$

Now let's examine the numerator in more detail:

$$5\times(3/5)=\frac{5}{1}\times\frac{3}{5}=\frac{15}{5}=3$$

Using this result, that the numerator is equal to three you can see that:

$$\frac{5\times(3/5)}{(g-3)}=\frac{3}{(g-3)}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

4) You use the same method to divide algebraic fractions as you would to divide numerical fractions. Firstly invert (turn upside down) the second fraction and then multiply instead of divide. You can use the same method explained in question 1 of this sheet to multiply the fractions.

After this you should **simplify** your answer, if you can, by identifying and **cancelling down** common factors in your numerator and denominator. If you have difficulty finding common factors, the study guide: *Simple Factorisation* can help.

a) $$\frac{3}{x}\div\frac{y}{7}=\frac{3}{x}\times\frac{7}{y}=\frac{21}{xy}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

b) $$\frac{y}{7}\div\frac{3}{x}=\frac{y}{7}\times\frac{x}{3}=\frac{xy}{21}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

c) $$\frac{x}{3}\div\frac{7}{y}=\frac{x}{3}\times\frac{y}{7}=\frac{xy}{21}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

d) $$\frac{7}{y}\div\frac{x}{3}=\frac{7}{y}\times\frac{3}{x}=\frac{21}{xy}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

e) $$\frac{a^3b^6c^4}{7} \div \frac{a^2b^6c^5}{x} = \frac{a^3b^6c^4}{7} \times \frac{x}{a^2b^6c^5} = \frac{a^3b^6c^4x}{7a^2b^6c^5}$$

There is a common factor of $a^2b^6c^4$ in the numerator and denominator which can be cancelled down to give:

$$\frac{a^3b^6c^4x}{7a^2b^6c^5} = \frac{ax \times \cancel{a^2b^6c^4}}{7c \times \cancel{a^2b^6c^4}} = \frac{ax}{7c}$$

f) $$\frac{K_0}{5A} \div \frac{K_0}{5A} = \frac{K_0}{5A} \times \frac{5A}{K_0} = \frac{5AK_0}{5AK_0} = 1$$

As the numerator and the denominator are the same, they cancel down to give 1.

g) $$5 \div \frac{z}{7} = \frac{5}{1} \div \frac{z}{7} = \frac{5}{1} \times \frac{7}{z} = \frac{35}{z}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

h) $$\frac{x}{2y} \div 4 = \frac{x}{2y} \div \frac{4}{1} = \frac{x}{2y} \times \frac{1}{4} = \frac{x}{8y}$$

This answer cannot be simplified any further as the numerator and denominator do not have any common factors.

i) $$\frac{4(p-4)^3}{4+s} \div \frac{(p-4)^4}{5+s} = \frac{4(p-4)^3}{(4+s)} \times \frac{(5+s)}{(p-4)^4} = \frac{4(5+s)(p-4)^3}{(4+s)(p-4)^4}$$

The addition of brackets helps you to see which cancellations are allowed and which are not. Here there is a common factor of $(p-4)^3$ in the numerator and denominator and so:

$$\frac{4(5+s)(p-4)^3}{(4+s)(p-4)^4} = \frac{4(5+s) \times \cancel{(p-4)^3}}{(4+s)(p-4) \times \cancel{(p-4)^3}} = \frac{4(5+s)}{(4+s)(p-4)}$$

5) In answering this question it is useful for you to remember that a fraction is just another way of writing a division of the numerator by the denominator.

a) $$1 \div \frac{1}{x} = \frac{1}{1} \times \frac{x}{1} = \frac{x}{1} = x$$

b) $\dfrac{1}{1/x} = 1 \div \dfrac{1}{x}$ which is the same as the previous question and so the answer is *x*.

c) $1 \div \dfrac{1}{1/x}$

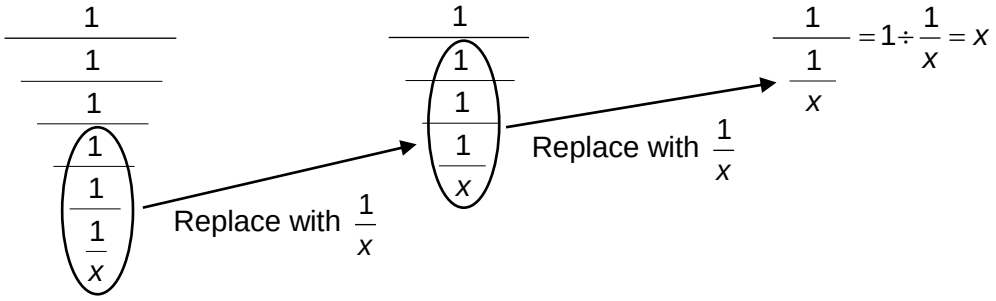
From the previous question you can see that $\dfrac{1}{1/x} = x$ and so:

$$1 \div \frac{1}{1/x} = 1 \div x = \frac{1}{x}$$

d) $\dfrac{1}{\dfrac{1}{\dfrac{1}{x}}}$

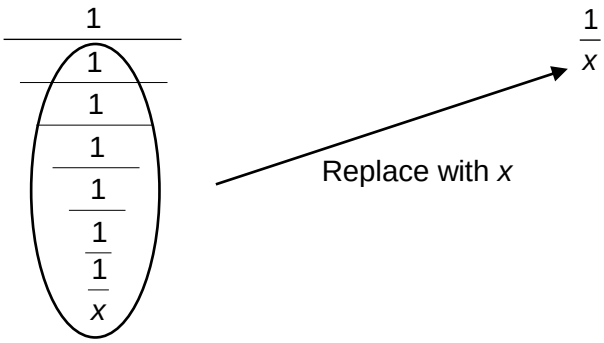
This can be rewritten as $1 \div \dfrac{1}{1/x}$ (count the number of 1's which appear in the question and its rewriting to help you see) which is the same as the previous question and so the answer is $\dfrac{1}{x}$.

e) Given the previous questions you may start to notice a pattern emerging which links that numbers of one in a number tower of this type and the result. However you can also use the shape of previous question to help. If you look at 5d you will notice that a tower with three one's over an *x* can be replaced with $\dfrac{1}{x}$. Using this, and the result from 5c:



And so a number tower of this type with six 1's is simply *x*.

f) Using the result from the previous question, that a number tower of this type with six 1's is equal to x you find that:



g) $x^{(-1)^5}$. Concentrating on the index, as $(-1)^5 = -1\times-1\times-1\times-1\times-1 = -1$:

$$x^{(-1)^5} = x^{-1} = \frac{1}{x}$$

In fact $x^{(-1)^n}$ is always $\frac{1}{x}$ when n is an odd number (can you see why?).

h) $x^{(-1)^6}$. Concentrating on the index, as $(-1)^6 = -1\times-1\times-1\times-1\times-1\times-1 = 1$:

$$x^{(-1)^6} = x^1 = x$$

In fact $x^{(-1)^n}$ is always x when n is an even number (can you see why?).