Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Steps into Algebra

Simple Factorisation

This guide introduces a simple technique to help the factorisation of algebraic expressions and equations with common factors in each term.

Introduction

Factorisation introduces **brackets** into mathematical expressions and equations. To do this you identify common elements within the terms of the expression or equation. These common elements are called **common factors**. Identifying common factors is useful when simplifying expressions and manipulating algebraic fractions. This guide is about the simplest form of factorisation where there are factors common to *every term* in an expression. The factorisation of **quadratic expressions** is different and is discussed in the study guide: *Factorising Quadratic Expressions*.

Factorising terms

Terms are pieces of mathematics which are separated by addition or subtraction symbols. For example the expression:

$$3x^2 + 5x - 6$$

has three terms $+3x^2$, $+5x$ and -6. Each term in an algebraic expression can be broken down into its constituent parts or **factors**. Normally a term will contain numbers and variables (in the form of letters). If the term only has variables in it remember that the variables are multiplied by 1. You could (and should) write the 1 before the variables to remind you that it is there, this will help when you are factorising.

Example: Write $6x^3$ in factored form.

To break down a term you first consider the sign of the term, then the numerical part and finally the alphabetical part (representing the variables) *in alphabetical order*.

Firstly consider the sign: the sign of $6x^3$ is positive.

Next consider the numerical part: the numerical part of $6x^3$ is 6. In **prime factor form** 6 can be written as $6 = 2 \cdot 3$.

Finally consider the variables: the variable part of $6x^3$ is x^3, which is $x \cdot x \cdot x$ in full.

Considering the three steps above, $6x^3$ can be written as $6x^3 = +2 \cdot 3 \cdot x \cdot x \cdot x$.

(If you find any of these steps difficult or confusing you should read the study guides on *SNALPHABET*, *Prime Factors* and/or *Laws of Indices*.)

Example: Write $-12x^2y^3$ in factored form.

Firstly the sign is negative. You should represent a minus sign as multiplication by -1.
The numerical part is 12 which is $2 \cdot 2 \cdot 3$ in prime factor form.
The alphabetical part written out in full is $x^2y^3 = x \cdot x \cdot y \cdot y \cdot y$.

So $-12x^2y^3$ can be written as $-12x^2y^3 = -1 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y$.

The method of simple factorisation

When you attempt to factorise an expression or equation you should follow this method:

1. List each term in the expression in factored form *down* your page.
2. Write an empty set of brackets as a template for your factorised answer.
3. Look at your factored forms, if *any* factor appears *in every list* then continue on to step 4. If not you *cannot* factorise your expression in this way.
4. Beginning with signs then numbers then variables in alphabetical order, check for factors which appear in every list (called **common factors**). These are written outside the brackets in your template and then crossed out from the lists. You can then multiply these common factors together to find the **overall** common factor of your expression.
5. The mathematics that is left in your lists after step 4 is then written as a sum inside the brackets in your template.
6. Tidy up your answer.

Example: Factorise $6x^3 - 12x^2y^3$.

Step 1: Write each term, $+6x^3$ and $-12x^2y^3$, in its factored form:

$$+6x^3 = +2\cdot 3\cdot x\cdot x\cdot x$$
$$-12x^2y^3 = -1\cdot 2\cdot 2\cdot 3\cdot x\cdot x\cdot y\cdot y\cdot y$$

Step 2: Your answer will take the form $?\,(\quad ?\quad)$.

Step 3: The factors 2, 3 and x all appear in both lists so you carry on to step 4.

Step 4: The first term is positive and the second term is negative, and so you do not have a common sign.
The numbers 2 and 3 are common in each list and so crossed from the lists.
Finally, you have $x\cdot x = x^2$ common in each list which is crossed from the lists.

$$+\cancel{2}\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot x$$
$$-1\cdot 2\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot y\cdot y\cdot y$$

You now can determine the common factor of your expression. You have 2 and 3 as numerical common factors, and x^2 as the variable common factor. Multiplying these together gives the common factor of the expression as $6x^2$ which is written outside the brackets in your template to give $6x^2(\quad ?\quad)$.

Step 5: You write what is left over from your lists as a sum inside the brackets.

In the first term $+x$ is left over.
In the second term you have $-1\cdot 2\cdot y\cdot y\cdot y = -2y^3$ left over.
So the factorised form is $6x^2(+x + -2y^3)$.

Step 6: The signs in the bracket need to be tidied up to give the final answer.

$$6x^2(+x + -2y^3) = 6x^2(x - 2y^3)$$

The result can always be checked by opening the brackets to make sure your answer is identical to the original expression (see study guide: *Opening Brackets*.)

Example: Factorise $xyz + x^2yz - xy^2z + xyz^2$.

Step 1: $xyz = +x\cdot y\cdot z$

$$x^2yz = +x\cdot x\cdot y\cdot z$$
$$-xy^2z = -1\cdot x\cdot y\cdot y\cdot z$$
$$xyz^2 = +x\cdot y\cdot z\cdot z$$

Step 2: Answer has the form $?\,(\quad ? \quad)$.

Step 3: x, y and z are common in each list and so you can carry on to step 4.

Step 4: Crossing out the common factor of xyz and writing it outside the bracket gives:

$$\begin{array}{l} +\cancel{x}\cdot\cancel{y}\cdot\cancel{z} \\ +\cancel{x}\cdot x\cdot\cancel{y}\cdot\cancel{z} \\ -1\cdot\cancel{x}\cdot\cancel{y}\cdot y\cdot\cancel{z} \\ +\cancel{x}\cdot\cancel{y}\cdot\cancel{z}\cdot z \end{array}$$

And an answer which looks like $xyz(\quad ? \quad)$.

Step 5: Write what is remaining from your lists as a sum inside the brackets. In the first term all the factors have been crossed out, remember that, $xyz = +1\cdot x\cdot y\cdot z$ and so you have +1 remaining. **If you cross out all the factors then there is always 1 left over, never 0.** In the second term you have $+x$, in the third term $-y$ and in the fourth term $+z$ remaining. The factorised form is $xyz(+1+x+-y+z)$.

Step 6: Tidying up the answer gives $xyz(+1+x+-y+z) = xyz(1+x-y+z)$.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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