Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Simple Factorisation

Simple Factorisation study guide

1i.

(a) $2x^2 - x^3 = x^2(2-x)$

First write each term, $+2x^2$ and $-x^3$, in its factored form:

$$+2x^2 = +2 \cdot x \cdot x$$
$$-x^3 = -x \cdot x \cdot x$$

The answer will be in the form ?(?).

The first term is positive and the second term is negative so there is not a common sign.

$x \cdot x = x^2$ is common in each list which is crossed from the lists.

$$+2 \cdot \cancel{x} \cdot \cancel{x}$$
$$-x \cdot \cancel{x} \cdot \cancel{x}$$

The common factor in this expression is x^2 which is written outside the brackets.

x^2(?)

Write what is left over as a sum inside the brackets.

First term: $+2$ is left over.

Second term: $-x$ is left over.

The factorised form is $x^2(+2+-x)$ and tidying the signs gives $x^2(2-x)$.

(b) $12x^2 + 6x^3 = 6x^2(2+x)$

Factorising each term gives:

$$+12x^2 = +2 \cdot 2 \cdot 3 \cdot x \cdot x$$
$$+6x^3 = +2 \cdot 3 \cdot x \cdot x \cdot x$$

Answer has the form ?(?)

The signs are common as are 2, 3 and x^2 so cross out the common factors:

$+\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}$

$+\cancel{2}\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot x$

and write them outside the brackets to give an answer, $+6x^2($? $)$

Write what is remaining as a sum inside the brackets.

First term: 2

Second term: x

Therefore the factorised form is $+6x^2(2+x)$ and tidying the signs gives $6x^2(2+x)$.

(c) $-2x^2-x^3=-x^2(2+x)$

Factorising each term gives:

$-2x^2=-2\cdot x\cdot x$

$-x^3=-x\cdot x\cdot x$

Answer has the form ?(?)

The signs are common as is x^2 so cross out the common factors: and write them outside the brackets to give:

$-\cancel{2}\cdot\cancel{x}\cdot\cancel{x}$

$-\cancel{x}\cdot\cancel{x}\cdot x$

and write them outside the brackets to give an answer, $-x^2($? $)$

Write what is remaining as a sum inside the brackets.

First term: 2

Second term: x

The factorised form: $-x^2(2+x)$, also this is the final answer as the signs are already tidied.

(d) $2x^2-4x^3=2x^2(1-2x)$

Factorising each term gives:

$+2x^2=+2\cdot x\cdot x$

$-4x^3=-2\cdot 2\cdot x\cdot x\cdot x$

Answer has the form ?(?)

The signs are not common but factors 2 and x^2 are so cross out the common factors:

$+\cancel{2}\cdot\cancel{x}\cdot\cancel{x}$

$-\cancel{2}\cdot 2\cdot\cancel{x}\cdot\cancel{x}\cdot x$

and write them outside the brackets to give an answer, $2x^2(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: $+1$

Second term: $-2x$

The factorised form is $2x^2(+1+-2x)$ and tidying the signs gives $2x^2(1-2x)$.

(e) $6x-12=6(x-2)$

Factorising each term gives:

and write them outside the brackets to give

$$+6x=+2\cdot 3\cdot x$$

$$-12=-2\cdot 2\cdot 3$$

Answer has the form $?(\quad ?\quad)$

The signs are not common but factors 2 and 3 are so cross out the common factors:

$$+\cancel{2}\cdot\cancel{3}\cdot x$$

$$-\cancel{2}\cdot 2\cdot\cancel{3}$$

and write them outside the brackets to give an answer, $6(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: $+x$

Second term: -2

So the factorised form is $6(+x+-2)$ and tidying the signs gives $6(x-2)$.

(f) $6-12x=6(1-2x)$

Factorising each term gives:

$$+6=+2\cdot 3$$

$$-12x=-2\cdot 2\cdot 3\cdot x$$

Answer has the form $?(\quad ?\quad)$

The signs are not common but factors 2 and 3 are so cross out the common factors:

$$+\cancel{2}\cdot\cancel{3}$$

$$-\cancel{2}\cdot 2\cdot\cancel{3}\cdot x$$

and write them outside the brackets to give an answer, $6(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: $+1$

Second term: $-2x$

So the factorised form is $6(+1+-2x)$ and tidying the signs gives $6(1-2x)$.

(g) $25x - 5y - 5 = 5(5x - y - 1)$

Factorising each term gives:

$+25x = +5 \cdot 5 \cdot x$

$-5y = -5 \cdot y$

$-5 = -5$

Answer has the form $?(\quad ?\quad)$

The factor 5 is common, cross this out:

$+\cancel{5} \cdot 5 \cdot x$

$-\cancel{5} \cdot y$

$-\cancel{5}$

and write outside the brackets to give an answer, $5(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: $+5x$

Second term: $-y$

Third term: -1

So the factorised form is $5(+5x + -y + -1)$ and tidying the signs gives $5(5x - y - 1)$.

(h) $\frac{1}{2}x + \frac{1}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$

Factorising each term gives:

$+\frac{1}{2}x = +\frac{1}{2} \cdot x$

$+\frac{1}{2} = +\frac{1}{2} \cdot \frac{1}{2}$

Answer has the form $?(\quad ?\quad)$

The signs are common as is $\frac{1}{2}$, cross out the common factors::

$+\cancel{\frac{1}{2}} \cdot x$

$+\cancel{\frac{1}{2}} \cdot \frac{1}{2}$

and write outside the brackets to give an answer, $+\frac{1}{2}(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: x

Second term: $\frac{1}{2}$

So the factorised form is: $+\frac{1}{2}(x + \frac{1}{2})$ and tidying the signs gives $\frac{1}{2}(x + \frac{1}{2})$

(i) $ab^3 - ab = ab(b^2 - 1)$

Factorising each term gives:

$+ab^3 = +a \cdot b \cdot b \cdot b$

$-ab = -a \cdot b$

Answer has the form ?(?)

The signs are not common but factors *a* and *b* are so cross out the common factors:

$+\cancel{a} \cdot \cancel{b} \cdot b \cdot b$

$-\cancel{a} \cdot \cancel{b}$

and write them outside the brackets to give an answer, *ab*(?)

Write what is remaining as a sum inside the brackets.

First term: $+b^2$

Second term: -1

So the factorised form is $ab(+b^2 + -1)$ and tidying the signs gives $ab(b^2 - 1)$.

(j) $5a - 10b + 15c = 5(a - 2b + 3c)$

Factorising each term gives:

$+5a = +5 \cdot a$

$-10b = -2 \cdot 5 \cdot b$

$+15c = +3 \cdot 5 \cdot c$

Answer has the form ?(?)

The factor 5 is common cross this out:

$+\cancel{5} \cdot a$

$-2 \cdot \cancel{5} \cdot b$

$+3 \cdot \cancel{5} \cdot c$

and write outside the brackets to give an answer, 5(?)

Write what is remaining as a sum inside the brackets.

First term: $+a$

Second term: $-2b$

Third term: $+3c$

So the factorised form is $5(+a + -2b + c)$ and tidying the signs gives $5(a - 2b + c)$.

2.

(a) $2xy^2 - x^3y = xy\left(2y - x^2\right)$

Factorising each term gives:

$$+2xy^2 = +2 \cdot x \cdot y \cdot y$$

$$-x^3y = -x \cdot x \cdot x \cdot y$$

Answer has the form ?(?)

The signs are not common but factors x and y are so cross out the common factors:

$$+2 \cdot \cancel{x} \cdot \cancel{y} \cdot y$$

$$-\cancel{x} \cdot x \cdot x \cdot \cancel{y}$$

and write them outside the brackets to give an answer, xy(?)

Write what is remaining as a sum inside the brackets.

First term: $+2y$

Second term: $-x^2$

So the factorised form is $xy(+2y + -x^2)$ and tidying the signs gives $xy(2y - x^2)$.

(b) $2xy^2z^3 - 6x^3yz + 4xyz$

Factorising each term gives:

$$+2xy^2z^3 = +2 \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z$$

$$-6x^3yz = -2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot z$$

$$+4xyz = +2 \cdot 2 \cdot x \cdot y \cdot z$$

Answer has the form ?(?)

The signs are not common but factors 2, x, y and z are, so cross out the common factors:

$$+\cancel{2} \cdot \cancel{x} \cdot \cancel{y} \cdot y \cdot \cancel{z} \cdot z \cdot z$$

$$-\cancel{2} \cdot 3 \cdot \cancel{x} \cdot x \cdot x \cdot \cancel{y} \cdot \cancel{z}$$

$$+\cancel{2} \cdot 2 \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{z}$$

and write them outside the brackets to give an answer, $2xyz$(?)

Write what is remaining as a sum inside the brackets.

First term: $+yz^2$

Second term: $-3x^2$

Third term: $+2$

So the factorised form is $2xyz(+yz^2 + -3x^2 + 2)$ and tidying the signs gives $2xyz(yz^2 - 3x^2 + 2)$.

(c) $12abc^3 - 6a^3b^2c + 4abc^2 = 2abc\left(6c^2 - 3a^2b + 2c\right)$

Factorising each term gives:

$+12abc^3 = +2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot c \cdot c \cdot c$

$-6a^3b^2c = -2 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c$

$+4abc^2 = +2 \cdot 2 \cdot a \cdot b \cdot c \cdot c$

Answer has the form ?(?)

The signs are not common but factors 2, *a*, *b* and *c* are, so cross out the common factors:

$+ 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot c \cdot c \cdot c$

$- 2 \cdot 3 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot c$

$+ 2 \cdot 2 \cdot a \cdot b \cdot c \cdot c$

and write them outside the brackets to give an answer, $2abc$(?)

Write what is remaining as a sum inside the brackets.

First term: $+6c^2$

Second term: $-3a^2b$

Third term: $+2c$

So the factorised form is $2abc(+6c^2 + -3a^2b + 2c)$ and tidying the signs gives $2abc(6c^2 - 3a^2b + 2c)$.

(d) $-3pqr - 6p^3q^2r^2 - 15pq^2r = -3pqr\left(1 + 2p^2qr + 5q\right)$

Factorising each term gives:

$-3pqr = -3 \cdot p \cdot q \cdot r$

$-6p^3q^2r^2 = -2 \cdot 3 \cdot p \cdot p \cdot p \cdot q \cdot q \cdot r \cdot r$

$-15pq^2r = -3 \cdot 5 \cdot p \cdot q \cdot q \cdot r$

Answer has the form ?(?)

The signs are common as are the factors 3, *p*, *q* and *r*, so cross out the common factors:

$- 3 \cdot p \cdot q \cdot r$

$- 2 \cdot 3 \cdot p \cdot p \cdot p \cdot q \cdot q \cdot r \cdot r$

$- 3 \cdot 5 \cdot p \cdot q \cdot q \cdot r$

and write them outside the brackets to give an answer, $-3pqr$(?)

Write what is remaining as a sum inside the brackets.

First term: 1

Second term: $2p^2qr$

Third term: $5q$

So the factorised form is $-3pqr(1+2p^2qr+5q)$.

3.

(a) $\dfrac{2xy^2}{3}+\dfrac{4xy}{9}=\dfrac{2xy}{3}\left(y+\dfrac{2}{3}\right)$

Factorising each term gives:

$$+\frac{2xy^2}{3}=+\frac{2}{3}\cdot x\cdot y\cdot y$$

$$+\frac{4xy}{9}=+\frac{2}{3}\cdot\frac{2}{3}\cdot x\cdot y$$

Answer has the form ?(?)

The signs are common as are $\frac{2}{3}$, x and y so cross out the common factors:

$$\cancel{+\tfrac{2}{3}}\cdot\cancel{x}\cdot\cancel{y}\cdot y$$

$$\cancel{+\tfrac{2}{3}}\cdot\tfrac{2}{3}\cdot\cancel{x}\cdot\cancel{y}$$

and write them outside the brackets to give an answer, $+\frac{2}{3}xy($?)

Write what is remaining as a sum inside the brackets.

First term: y

Second term: $\frac{2}{3}$

So the factorised form is $+\frac{2}{3}xy\left(y+\frac{2}{3}\right)$ and tidying the signs gives $\frac{2}{3}xy\left(y+\frac{2}{3}\right)$.

(b) $\frac{2}{3}xy^2+\frac{4}{9}xy=\frac{2}{3}xy\left(y+\frac{2}{3}\right)$

This question is identical to the previous question apart from the algebraic fractions are written slightly differently.

(c) $\dfrac{4}{x}+\dfrac{8}{x^2}=\dfrac{4}{x}\left(1+\dfrac{4}{x}\right)$

Factorising each term gives:

$$+\frac{4}{x}=+2\cdot 2\cdot\frac{1}{x}$$

$$+\frac{8}{x^2}=+2\cdot 2\cdot 2\cdot\frac{1}{x}\cdot\frac{1}{x}$$

Answer has the form ?(?)

The signs are common as are $2\cdot 2$ and $\frac{1}{x}$ so cross out the common factors:

$$\cancel{+2\cdot 2}\cdot\cancel{\tfrac{1}{x}}$$

$+\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{\frac{1}{x}}\cdot\frac{1}{x}$

and write them outside the brackets to give an answer, $+\frac{4}{x}(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: 1

Second term: $\frac{2}{x}$

So the factorised form is $+\frac{4}{x}\left(1+\frac{2}{x}\right)$ and tidying the signs gives $\frac{4}{x}\left(1+\frac{2}{x}\right)$.

(d) $\dfrac{15r^2}{pq}+\dfrac{5r}{pq^2}$

Factorising each term gives:

$$+\frac{15r^2}{pq}=+3\cdot 5\cdot r\cdot r\cdot\frac{1}{p}\cdot\frac{1}{q}$$

$$+\frac{5r}{pq^2}=+5\cdot r\cdot\frac{1}{p}\cdot\frac{1}{q}\cdot\frac{1}{q}$$

Answer has the form $?(\quad ?\quad)$

The signs are common as are 5, r, $\frac{1}{p}$ and $\frac{1}{q}$ so cross out the common factors:

$$+\cancel{3}\cdot\cancel{5}\cdot\cancel{r}\cdot r\cdot\cancel{\frac{1}{p}}\cdot\cancel{\frac{1}{q}}$$

$$+\cancel{5}\cdot\cancel{r}\cdot\cancel{\frac{1}{p}}\cdot\cancel{\frac{1}{q}}\cdot\frac{1}{q}$$

and write them outside the brackets to give an answer, $+\dfrac{5r}{pq}(\quad ?\quad)$

Write what is remaining as a sum inside the brackets.

First term: $3r$

Second term: $\frac{1}{q}$

So the factorised form is $+\dfrac{5r}{pq}\left(3r+\dfrac{1}{q}\right)$ and tidying the signs gives $\dfrac{5r}{pq}\left(3r+\dfrac{1}{q}\right)$.

(e) $\dfrac{1}{s}-\dfrac{1}{s^2}=\dfrac{1}{s}\left(1-\dfrac{1}{s}\right)$

Factorising each term gives:

$$+\frac{1}{s}=+\frac{1}{s}$$

$$-\frac{1}{s^2}=-\frac{1}{s}\cdot\frac{1}{s}$$

Answer has the form $?(\quad ?\quad)$

The signs are not common but the factor $\frac{1}{s}$ is, so cross out the common factors:

$$+\cancel{\frac{1}{s}}$$

$$-\cancel{\frac{1}{s}}\cdot\frac{1}{s}$$

and write them outside the brackets to give an answer, $\frac{1}{s}($? $)$

Write what is remaining as a sum inside the brackets.

First term: $+1$

Second term: $-\frac{1}{s}$

So the factorised form is $\frac{1}{s}\left(+1+-\frac{1}{s}\right)$ and tidying the signs gives $\frac{1}{s}\left(1-\frac{1}{s}\right)$

(f) $$\frac{8xz^2}{7y}+\frac{4xz^3}{49y^2}=\frac{4xz^2}{7y}\left(2z+\frac{z}{7y}\right)$$

Factorising each term gives:

$$+\frac{8xz^2}{7y}=+2\cdot 2\cdot 2\cdot x\cdot z\cdot z\cdot\frac{1}{7}\cdot\frac{1}{y}$$

$$+\frac{4xz^3}{49y^2}=+2\cdot 2\cdot x\cdot z\cdot z\cdot z\cdot\frac{1}{7}\cdot\frac{1}{7}\cdot\frac{1}{y}\cdot\frac{1}{y}$$

Answer has the form ?(?)

The signs are common as are 2, 2, x, z, z, $\frac{1}{7}$ and $\frac{1}{y}$ so cross out the common factors:

$$+\cancel{2}\cdot\cancel{2}\cdot 2\cdot\cancel{x}\cdot\cancel{z}\cdot\cancel{z}\cdot\cancel{\frac{1}{7}}\cdot\cancel{\frac{1}{y}}$$

$$+\cancel{2}\cdot\cancel{2}\cdot\cancel{x}\cdot\cancel{z}\cdot\cancel{z}\cdot z\cdot\cancel{\frac{1}{7}}\cdot\frac{1}{7}\cdot\cancel{\frac{1}{y}}\cdot\frac{1}{y}$$

and write them outside the brackets to give an answer, $+\frac{4xz^2}{7y}($? $)$

Write what is remaining as a sum inside the brackets.

First term: 2

Second term: $\frac{z}{7y}$

So the factorised form is $+\frac{4xz^2}{7y}\left(2+\frac{z}{7y}\right)$ and tidying the signs gives $\frac{4xz^2}{7y}\left(2+\frac{z}{7y}\right)$.