Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Laws of Indices

Laws of Indices study guide

1. Follow each step carefully identifying which laws have been used at each step.

(a) $2^6 2^{-4} = 2^{6-4} = 2^2 = 4$

(b) $9^{1/2}3^2 = \left(3^2\right)^{1/2}3^2 = 3^{2\times(1/2)}3^2 = 3^1 3^2 = 3^{1+2} = 3^3 = 27$

(c) $64^{1/3} = \sqrt[3]{64} = 4$ since $4\times 4\times 4 = 64$.

(d) $4^{1/2}8^{1/3} = \left(2^2\right)^{1/2}\left(2^3\right)^{1/3} = 2^{2\times(1/2)}2^{3\times(1/3)} = 2^1 2^1 = 2^{1+1} = 2^2 = 4$

(e) $\dfrac{3^3}{3^5} = 3^{3-5} = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

(f) $\dfrac{4^{1/2}}{16^{1/4}} = \dfrac{\left(2^2\right)^{1/2}}{\left(2^4\right)^{1/4}} = \dfrac{2^1}{2^1} = 2^{1-1} = 2^0 = 1$

(g) $\dfrac{2^0}{2^{-2}} = 2^{0-(-2)} = 2^2 = 4$

(h) $\dfrac{5^{-1}}{25^{-1/2}} = \dfrac{5^{-1}}{\left(5^2\right)^{-1/2}} = \dfrac{5^{-1}}{5^{-1}} = 5^{-1-(-1)} = 5^{-1+1} = 5^0 = 1$

2. Here a dot is used to imply multiplication to avoid confusion.

(a) $x^2 x^8 = x^{2+8} = x^{10}$

(b) $x^2 x^{-8} = x^{2+(-8)} = x^{-6}$

(c) $x^{-2} x^{-8} = x^{-2+(-8)} = x^{-10}$

(d) $\left(x^2\right)^{-8} = x^{2\cdot(-8)} = x^{-16}$

(e) $\dfrac{x^{-2}}{x^8} = x^{-2-8} = x^{-10}$

(f) $\dfrac{x^2}{x^{-8}} = x^{2-(-8)} = x^{10}$

(g) $\dfrac{x^2}{x^8} = x^{2-8} = x^{-6}$

(h) $\left(\dfrac{1}{x^8}\right)^{-2} = \left(x^{-8}\right)^{-2} = x^{(-8)\cdot(-2)} = x^{16}$

So $a = f$, $b = g$ and $c = e$.

3.

(a) $\left(\frac{1}{2}\right)^{-2} = \left(2^{-1}\right)^{-2} = 2^{(-1)(-2)} = 2^2 = 4$

(b) $\dfrac{1}{2^{-1}} = \left(2^{-1}\right)^{-1} = 2^1 = 2$

(c) $27^{-1/3} = \dfrac{1}{27^{1/3}} = \dfrac{1}{\sqrt[3]{27}} = \dfrac{1}{3}$

(d) $\left(\dfrac{16}{49}\right)^{1/2} = \dfrac{16^{1/2}}{49^{1/2}} = \dfrac{\sqrt{16}}{\sqrt{49}} = \dfrac{4}{7}$

4.

(a) $$\frac{y^{1/6}y^{-2/3}}{y^{1/4}} = \frac{y^{(1/6)-(2/3)}}{y^{1/4}} = \frac{y^{(1/6)-(4/6)}}{y^{1/4}} = \frac{y^{-3/6}}{y^{1/4}} = y^{-(1/2)-(1/4)} = y^{-3/4}$$

(b) $$\frac{p^{1/2}p^{-3/4}}{p^{-1/4}} = \frac{p^{(1/2)-(3/4)}}{p^{-1/4}} = \frac{p^{-1/4}}{p^{-1/4}} = 1$$

(c) $$\frac{\sqrt{x}\sqrt{x^3}}{x^{-3}} = \frac{x^{1/2}\left(x^3\right)^{1/2}}{x^{-3}} = \frac{x^{1/2}x^{3/2}}{x^{-3}} = \frac{x^{(1/2)+(3/2)}}{x^{-3}} = \frac{x^2}{x^{-3}} = x^{2-(-3)} = x^5$$

(d) $$\frac{\left(\sqrt{t}\right)^3 t^2}{\sqrt{\left(t^5\right)}} = \frac{\left(t^{1/2}\right)^3 t^2}{\left(t^5\right)^{1/2}} = \frac{t^{3/2}t^2}{t^{5/2}} = \frac{t^{(3/2)+2}}{t^{5/2}} = \frac{t^{7/2}}{t^{5/2}} = t^{(7/2)-(5/2)} = t^1 = t$$

5. A good tactic in these types of questions is to recognise squares and cubes and so on, to deal with the roots.

(a) Use $9 = 3^2$ and $8 = 2^3$ so:

$$\frac{9^{1/2}8^{1/2}}{2^{1/2}} = \frac{\left(3^2\right)^{1/2}\left(2^3\right)^{1/2}}{2^{1/2}} = \frac{3^1 2^{3/2}}{2^{1/2}} = 3 \times \left(2^{(3/2)-(1/2)}\right) = 3 \times 2 = 6$$

(b) Use $25 = 5^2$ and $125 = 5^3$ so:

$$\frac{5^{1/2}5^0 25^{1/2}}{125^{1/3}} = \frac{5^{(1/2)+0}\left(5^2\right)^{1/2}}{\left(5^3\right)^{1/3}} = \frac{5^{1/2}5^1}{5^1} = 5^{(1/2)+1-1} = 5^{1/2}$$

(c) Use $8 = 2^3$, $16 = 2^4$ and $32 = 2^5$ so:

$$\frac{8^{1/3}16^{1/3}}{32^{-1/3}} = \frac{\left(2^3\right)^{1/3}\left(2^4\right)^{1/3}}{\left(2^5\right)^{-1/3}} = \frac{2^1 2^{4/3}}{2^{-5/3}} = 2^{1+(4/3)-(-5/3)} = 2^4 = 16$$

(d) Use $9 = 3^2$ and $27 = 3^3$ so:

$$\frac{9^{1/3}27^{-1/2}}{3^{-1/6}3^{-2/3}} = \frac{\left(3^2\right)^{1/3}\left(3^3\right)^{-1/2}}{3^{(-1/6)+(-2/3)}} = \frac{3^{2/3}3^{-3/2}}{3^{-5/6}} = 3^{(2/3)-(3/2)-(-5/6)} = 3^0 = 1$$

6. A good tactic here is to deal with each letter individually and in alphabetical order:

(a) $$\frac{a^2b^{-4}c^5}{ab^{-7}c^2} = a^{2-1}b^{-4-(-7)}c^{5-2} = ab^3c^3$$

(b) $$\sqrt{\frac{ab^6c^{-3}}{a^{-3}b^{-2}c}} = \left(a^{1-(-3)}b^{6-(-2)}c^{-3-1}\right)^{1/2} = \left(a^4b^8c^{-4}\right)^{1/2} = a^2b^4c^{-2} = \frac{a^2b^4}{c^2}$$

(c) $$\frac{ab^{1/6}c^{-2/3}}{a^2b^{-1/4}c^{-4/5}} = a^{1-2}b^{(1/6)-(-1/4)}c^{(-2/3)-(-4/5)} = a^{-1}b^{5/12}c^{2/15} = \frac{b^{5/12}c^{2/15}}{a}$$

7. Here a dot is used to imply multiplication to avoid confusion.

(a) $$\left(p^2q^{1/2}r^5\right)^3 = p^{2\cdot3}q^{(1/2)\cdot3}r^{5\cdot3} = p^6q^{3/2}r^{15}$$

(b) $$\left(\frac{p^{-2}r^3}{q^6}\right)^3 = \frac{p^{-2\cdot3}r^{3\cdot3}}{q^{6\cdot3}} = \frac{p^{-6}r^9}{q^{18}} = \frac{r^9}{p^6q^{18}}$$

(c) $$\left(p^4q^8r^{-6}\right)^{-1/2} = p^{-2}q^{-4}r^3 = \frac{r^3}{p^2q^4}$$