Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Adding and Subtracting Fractions

Adding and Subtracting Fractions study guide

1. When the fractions both have the same denominator (bottom) you can do them using just simple adding and subtracting. Or some of them these can be done by adding and subtracting fractions that you already know. Try saying them out loud (or in your head if that's not practical)

(a) $\frac{2}{3}$

"A third plus a third is two thirds" or, if you want to do it more formally, the denominators (bottoms) are both 3 and so you can just add the numerators (tops) to give $1+1=2$ and so the new numerator is 2. Pictorially this looks like:

(b) 1

"A third plus two thirds is three thirds which is a whole or, in other words, one" Again the denominators are the same and so you just add the numerators to give $1+2=3$. Then you have:

$$\frac{1}{3}+\frac{2}{3}=\frac{3}{3}=1$$

It is best not to leave it as $3/3$ because this can be cancelled down to 1 which is much simpler. Pictorially this looks like:

(c) $\frac{3}{4}$

Say it out loud or in your head: “A half plus a quarter is three quarters”

(d) $\frac{1}{4}$

“A half minus a quarter is a quarter” You know that two quarters make a half and so, if you take a quarter from a half, you are left with just one quarter.

(e) 1

“Five sixths plus one sixth is six sixths which is a whole or 1” Formally you may write

$$\frac{5}{6}+\frac{1}{6}=\frac{6}{6}=1$$

(f) $\frac{4}{3}$ or $1\frac{1}{3}$

“One sixths plus seven sixths is eight sixths” So:

$$\frac{1}{6}+\frac{7}{6}=\frac{8}{6}$$

Then you can cancel this down by dividing the top and bottom by 2 to get $4/3$. It is preferable to write it as an improper fraction as $4/3$ although the mixed fraction $1\frac{1}{3}$ is also common. The important thing to remember with mixed fractions is that, even though the numbers are written next to each other, it is **not** $1\times\frac{1}{3}$ but $1+\frac{1}{3}$.

(g) $-\frac{1}{3}$

"Minus a sixth minus a sixth is minus two sixths which cancels down to minus a third" You need to be clear that a negative number minus another negative number is also a negative number. Then:

$$-\frac{1}{6}-\frac{1}{6}=-\frac{2}{6}=-\frac{1}{3}$$

(h) 0

"An eighth minus an eighth is zero" Anything minus itself is 0.

(i) 0

"Minus an eight plus an eight is zero". This question is just the same as the previous one but with the negative number before the positive one.

2. Now you have to find a common denominator by multiplying the denominators of the two fractions.

Remember you can only add or subtract the numerators of fractions if they have the same denominator.

a) $\frac{7}{6}$ or $1\frac{1}{6}$

You get the common denominator by multiplying the two denominators so $3\times 2=6$

Then, to get 6 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 2 so that:

$$\frac{2}{3}\times\frac{2}{2}=\frac{4}{6}$$

So $2/3$ and $4/6$ are **equivalent fractions**. To get 6 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 3 so that:

$$\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$$

So $1/2$ and $3/6$ are equivalent fractions. Now the denominators are the same in both fractions, you can add the numerators to give:

$$\frac{4}{6}+\frac{3}{6}=\frac{7}{6}$$

which is the preferred form for the answer although you may also see it written as a mixed fraction as $1\frac{1}{6}$.

b) $-\frac{1}{6}$

You get the common denominator by multiplying the two denominators so $3 \times 2 = 6$. Then, to get 6 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 3 so that:

$$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$$

So $1/2$ and $3/6$ are equivalent fractions. To get 6 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 2 so that:

$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$

So $2/3$ and $4/6$ are equivalent fractions. Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{3}{6} - \frac{4}{6} = -\frac{1}{6}$$

c) $\frac{13}{14}$

You get the common denominator by multiplying the two denominators so $7 \times 2 = 14$. Then, to get 14 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 2 so that:

$$\frac{3}{7} \times \frac{2}{2} = \frac{6}{14}$$

So $3/7$ and $6/14$ are equivalent fractions. To get 14 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 7 so that:

$$\frac{1}{2} \times \frac{7}{7} = \frac{7}{14}$$

Now the denominators are the same in both fractions, you can add the numerators to give:

$$\frac{6}{14} + \frac{7}{14} = \frac{13}{14}$$

d) $\frac{7}{12}$

You get the common denominator by multiplying the two denominators so $3 \times 4 = 12$. Then, to get 12 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 4 so that:

$$\frac{4}{3}\times\frac{4}{4}=\frac{16}{12}$$

So $4/3$ and $16/12$ are equivalent fractions. To get 12 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 3 so that:

$$\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$$

So $3/4$ and $9/12$ are equivalent fractions. Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{16}{12}-\frac{9}{12}=\frac{7}{12}$$

e) $\frac{124}{105}$ or $1\frac{19}{105}$

You get the common denominator by multiplying the two denominators so $15\times 7=105$. Then, to get 105 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 7 so that:

$$\frac{7}{15}\times\frac{7}{7}=\frac{49}{105}$$

To get 105 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 15 so that:

$$\frac{5}{7}\times\frac{15}{15}=\frac{75}{105}$$

Now the denominators are the same in both fractions, you can add the numerators to give:

$$\frac{49}{105}+\frac{75}{105}=\frac{124}{105}$$

which is the preferred form for the answer although you may also see it written as a mixed fraction as $1\frac{19}{105}$.

f) $\frac{103}{300}$

You get the common denominator by multiplying the two denominators so $3\times 100=300$. Then, to get 300 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 100 so that:

$$\frac{1}{3}\times\frac{100}{100}=\frac{100}{300}$$

To get 100 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 3 so that:

$$\frac{1}{100}\times\frac{3}{3}=\frac{3}{300}$$

Now the denominators are the same in both fractions, you can add the numerators to give:

$$\frac{100}{300}+\frac{3}{300}=\frac{103}{300}$$

g) $-\frac{379}{700}$

You get the common denominator by multiplying the two denominators so $100\times 7=700$. Then, to get 700 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 7 so that:

$$\frac{3}{100}\times\frac{7}{7}=\frac{21}{700}$$

To get 700 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 100 so that:

$$\frac{4}{7}\times\frac{100}{100}=\frac{400}{700}$$

Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{21}{700}-\frac{400}{700}=-\frac{379}{700}$$

h) $\frac{14}{99}$

You get the common denominator by multiplying the two denominators so $11\times 9=99$. Then, to get 99 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 9 so that:

$$\frac{4}{11}\times\frac{9}{9}=\frac{36}{99}$$

To get 99 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 11 so that:

$$\frac{2}{9}\times\frac{11}{11}=\frac{22}{99}$$

Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{36}{99}-\frac{22}{99}=\frac{14}{99}$$

i) $-\frac{1457}{543000}$

You get the common denominator by multiplying the two denominators so $1000 \times 543 = 543000$. Then, to get 543000 to be the denominator of the first fraction, you multiply both the numerator and the denominator by 543 so that:

$$\frac{1}{1000} \times \frac{543}{543} = \frac{543}{543000}$$

To get 543000 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 1000 so that:

$$\frac{2}{543} \times \frac{1000}{1000} = \frac{2000}{543000}$$

Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{543}{543000} - \frac{2000}{543000} = -\frac{1457}{543000}$$

3. In these ones you could just multiply the denominators but it is more efficient to find a lower number that has both the denominators as factors. This is known as **the lowest common multiple** (see study guide: *Lowest Common Multiple*)

a) $\frac{3}{4}$

Remember "a half plus a quarter is three quarters". To do it more formally you find a common denominator. Now you could multiply the denominators to get $2 \times 4 = 8$ and then convert the fractions to have this denominator which would give

$$\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$$

And then you can cancel this down to get $3/4$.

It is a bit quicker and neater to choose a lower common denominator which is 4 since 4 has both the original denominators (2 and 4) as factors. Then you can convert the first fraction by multiplying both the numerator and the denominator by 2 so that:

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

Then you have:

$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$

and there is no need for any cancelling down.

b) $\frac{1}{2}$

Again, you could choose a common denominator of $6 \times 3 = 18$ but it is quicker to use 6 since 3 is a factor of 6. Convert the second fraction by multiplying both the numerator and the denominator by 2 so that:

$$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$$

Then you have:

$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$

which cancels down to $1/2$.

c) $\frac{5}{8}$

Since 4 is a factor of 8, the lowest common multiple is 8. Convert the second fraction by multiplying both the numerator and the denominator by 2 so that:

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

Then you have:

$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

d) $\frac{4}{9}$

Since 3 is a factor of 9, the lowest common multiple is 9. Convert the second fraction by multiplying both the numerator and the denominator by 3 so that:

$$\frac{1}{3} \times \frac{3}{3} = \frac{3}{9}$$

Then you have:

$$\frac{7}{9} - \frac{3}{9} = \frac{4}{9}$$

e) $\frac{7}{12}$

This is slightly different since 4 is not a factor of 6. However 24 is not the lowest common multiple. It is 12 because 12 has both 6 and 4 as factors. See the study guide *Lowest Common Multiple* for help with this. Convert the first fraction by multiplying both the numerator and the denominator by 3 so that:

$$\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$$

and convert the second fraction by multiplying both the numerator and the denominator by 2 so that:

$$\frac{1}{6}\times\frac{2}{2}=\frac{2}{12}$$

Then you have:

$$\frac{9}{12}-\frac{2}{12}=\frac{7}{12}$$

f) $-\frac{1}{12}$

Since 2 is a factor of 12, the lowest common multiple is 12. Convert the second fraction by multiplying both the numerator and the denominator by 6 so that:

$$\frac{1}{2}\times\frac{6}{6}=\frac{6}{12}$$

Then you have:

$$\frac{5}{12}-\frac{6}{12}=-\frac{1}{12}$$

g) $\frac{5}{6}$

Since 4 is a factor of 12, the lowest common multiple is 12. Convert the first fraction by multiplying both the numerator and the denominator by 3 so that:

$$\frac{3}{4}\times\frac{3}{3}=\frac{9}{12}$$

Then you have:

$$\frac{9}{12}+\frac{1}{12}=\frac{10}{12}$$

which cancels down to $\frac{5}{6}$.

h) $-\frac{1}{28}$

In this one 4 is not a factor of 14. However $14\times4=56$ is not the lowest common multiple. It is 28 because 28 has both 4 and 14 as factors. Convert the first fraction by multiplying both the numerator and the denominator by 2 so that:

$$\frac{3}{14}\times\frac{2}{2}=\frac{6}{28}$$

and convert the second fraction by multiplying both the numerator and the denominator by 7 so that:

$$\frac{1}{4}\times\frac{7}{7}=\frac{7}{28}$$

Then you have:

$$\frac{6}{28}-\frac{7}{28}=-\frac{1}{28}$$

i) $\frac{19}{36}$

In this one 9 is not a factor of 12. However $9\times12=108$ is not the lowest common multiple. It is 36 because 36 has both 9 and 12 as factors. Convert the first fraction by multiplying both the numerator and the denominator by 4 so that:

$$\frac{4}{9}\times\frac{4}{4}=\frac{16}{36}$$

and convert the second fraction by multiplying both the numerator and the denominator by 3 so that:

$$\frac{1}{12}\times\frac{3}{3}=\frac{3}{36}$$

Then you have:

$$\frac{16}{36}+\frac{3}{36}=\frac{19}{36}$$

4. First convert both mixed fractions to improper fractions by multiplying whole number by the denominator of the fractional part and then adding this to the numerator. Then apply the same methods as before. Remember that in a mixed fraction the whole number is **not** multiplying the fraction part even though they are written next to each other. They are actually being added.

a) $\frac{7}{6}$

Remember the mixed fraction $1\frac{1}{2}$ does **not** mean $1\times\frac{1}{2}$ but is actually the same as saying, "two halves plus one half" or $\frac{2}{2}+\frac{1}{2}$ which is "three halves" or $3/2$. This is the most common mixed fraction and so it is worth remembering the improper form which is, in general, more useful. Now you have converted it to an improper fraction, you can use the same methods as before. The denominators are 2 and 3 which are co-prime and so you get the common denominator by multiplying them so $3\times2=6$. Then, to get 6 to be the denominator of the

first fraction, you multiply both the numerator and the denominator of the improper fraction by 3 so that:

$$\frac{3}{2}\times\frac{3}{3}=\frac{9}{6}$$

To get 6 to be the denominator of the second fraction, you multiply both the numerator and the denominator by 2 so that:

$$\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$$

Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{9}{6}-\frac{2}{6}=\frac{7}{6}$$

which is the preferred form for the answer although you may also see it written as a mixed fraction as $1\frac{5}{6}$.

b) $\frac{9}{2}$

The mixed fraction $5\frac{1}{3}$ is the same as saying, “fifteen thirds plus one third” or $\frac{15}{3}+\frac{1}{3}$ which is “sixteen thirds” or $16/3$. Now you have converted it to an improper fraction, you can use the same methods as before. The denominators are 3 and 6. Since 3 is a factor of 6, 6 is the lowest common multiple. Then, to get 6 to be the denominator of the first fraction, you multiply both the numerator and the denominator of the improper fraction by 2 so that:

$$\frac{16}{3}\times\frac{2}{2}=\frac{32}{6}$$

Now the denominators are the same in both fractions, you can subtract the numerators to give:

$$\frac{32}{6}-\frac{5}{6}=\frac{27}{6}=\frac{9}{2}$$

which is the preferred form for the answer although you may also see it written as a mixed fraction as $4\frac{1}{2}$.

c) $\frac{55}{8}$

The mixed fraction $4\frac{3}{8}$ is the same as $\frac{32}{8}+\frac{3}{8}=\frac{35}{8}$ and the mixed fraction $2\frac{1}{4}$ is the same as $\frac{9}{4}+\frac{1}{4}=\frac{10}{4}$. The denominators are 8 and 4. Since 4 is a factor of 8, 8 is the lowest common multiple. Then, to get 8 to be the denominator of the second fraction, you multiply both the numerator and the denominator of the improper fraction by 2 so that:

$$\frac{10}{4}\times\frac{2}{2}=\frac{20}{8}$$

Now the denominators are the same in both fractions, you can add the numerators to give:

$$\frac{35}{8}+\frac{20}{8}=\frac{55}{8}$$

which is the preferred form for the answer although you may also see it written as a mixed fraction as $6\frac{7}{8}$.

5. The same techniques apply to adding three or more fractions. Just make sure that the denominator has all three (or more) fractions as a factor.

a) 1

Since 2 and 3 are both factors of 6. The lowest common multiple is 6. $1/2$ can be written as $3/6$ and $1/3$ can be written as $2/6$ so that: the addition becomes

$$\frac{3}{6}+\frac{2}{6}+\frac{1}{6}=\frac{6}{6}$$

which simplifies to 1.

b) $\frac{2}{3}$

This can be rearranged to:

$$\frac{1}{2}-\frac{1}{2}+\frac{2}{3}$$

and so the first two fractions cancel to leave $2/3$; even though there are fractions, the rules of addition and subtraction still apply.

c) $\frac{9}{20}$

The lowest common multiple of the denominators is 20.

$\frac{3}{4}$ can be written as $\frac{15}{20}$

$\frac{1}{5}$ can be written as $\frac{4}{20}$ and

$\frac{1}{10}$ can be written as $\frac{2}{20}$ and

So the answer is

$$\frac{15}{20}-\frac{4}{20}-\frac{2}{20}=\frac{9}{20}$$

d) 0.0001001

You have to realise that $10^7 = 10^4 \cdot 10^3$ Then, since 10^4 is a factor of 10^7 the lowest common multiple is 10^7 and

$\frac{1}{10^4}$ can be written as $\frac{10^3}{10^7}$

So the answer is

$$\frac{10^3}{10^7}+\frac{1}{10^7}=\frac{10^3+1}{10^7}=\frac{1001}{10000000}$$

So to get the decimal answer, you just move the decimal point seven places to the left.

e) 0.111

Since both 10 and 100 are factors of 1000. 1000 is the lowest common multiple.

$\frac{1}{10}$ can be written as $\frac{100}{1000}$ and

$\frac{1}{100}$ can be written as $\frac{10}{1000}$

So the answer is

$$\frac{100}{1000}+\frac{10}{1000}+\frac{1}{1000}=\frac{111}{1000}$$

which is 0.111 as a decimal.

f) $0.\dot{1}$ 0.1 recurring

This is an example of an infinite series. Even though the series goes on forever, it still has a number that it sums up to.

This is a generalisation of the previous example. You can choose any power of 10 to be the denominator such as 100,000 or 100,000,000. However large you make it the numerator will always be 1's and so the decimal will be 0.1 recurring which is the same as $\frac{1}{9}$

g) $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$

See question 5 a) above.

Then, to get a general unit fraction $1/a$, you can divide both sides of this equation by *a* (or, in other words, multiply by $1/a$) so that:

$$\frac{1}{a}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right)=1\cdot\frac{1}{a}$$ and so

$$\frac{1}{a}\cdot\frac{1}{2}+\frac{1}{a}\cdot\frac{1}{3}+\frac{1}{a}\cdot\frac{1}{6}=\frac{1}{a}$$ and then the answer is

$$\frac{1}{2a}+\frac{1}{3a}+\frac{1}{6a}=\frac{1}{a}$$

This is an example of an Egyptian fraction.