Learning Enhancement Team

Steps into Numeracy

Highest Common Factor

This guide introduces the highest common factor of lists of numbers (usually pairs) and techniques for finding it.

What is a highest common factor?

The **highest common factor** (HCF) of a list of numbers is the largest number that divides wholly into *all* the numbers. In **number theory** the HCF is called the **greatest common divisor** (GCD). Working out the HCF is beneficial when factorising expressions and/or simplifying fractions. In order to fully understand the methods in this guide you should be able to factorise a number into its **prime factors** (see study guide: *Prime Factors*).

Methods

It is clearer to learn methods for finding highest common factors with pairs of numbers rather than with long lists of numbers.

For smaller numbers, you can calculate the HCF by following this procedure:

(1)   Write a list of all the factors (from smallest to largest) of each number in turn.

(2)   Look at the lists, the largest number appearing in both lists is the HCF of the two numbers.

Example: What is the highest common factor of 12 and 18?

The factors of 12 are   1   2   3   4   **6**   12
The factors of 18 are   1   2   3   **6**   9   18

As you can see 6 is the highest number that appears in both lists so $\text{HCF}(12,18) = 6$.

For larger numbers, finding the highest common factor using the method above is not so easy. You can use the following method to find the HCF in these situations.

(1)   Write the numbers in *prime factor* form.

(2)   Match up the appearances of prime factors in each list and delete factors which have no match.

(3) Multiply the remaining factors in any one list to find the highest common factor. It should not matter which list you pick.

Example: What is the HCF of 240 and 924?

(1) The prime factor form of 240 is $2 \times 2 \times 2 \times 2 \times 3 \times 5$
The prime factor form of 924 is $2 \times 2 \times 3 \times 7 \times 11$

(2) Matching up and deleting

$$\cancel{2 \times 2} \times 2 \times 2 \times 3 \times \cancel{5}$$
$$2 \times 2 \times 3 \times \cancel{7 \times 11}$$
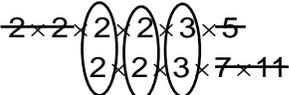
(3) So the HCF of 240 and 924 is $2 \times 2 \times 3 = 12$ which is written as HCF $(240, 924) = 12$.

Using highest common factors

You can use highest common factors to simplify fractions.

Example: Write $\dfrac{240}{924}$ in its simplest form.

From the previous example: $240 = 12 \times 2 \times 2 \times 5 = 12 \times 20$ and $924 = 12 \times 7 \times 11 = 12 \times 77$

Cancellation of the HCF gives the fraction in its simplest form $\dfrac{240}{924} = \dfrac{\cancel{12} \times 20}{\cancel{12} \times 77} = \dfrac{20}{77}$.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

- ☏ Call: 01603 592761
- ▬ Ask: ask.let@uea.ac.uk
- ☝ Click: https://portal.uea.ac.uk/student-support-service/learning-enhancement

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