University of East Anglia
STUDENT SUPPORT SERVICE

Prime Factors

This guide introduces the common mathematical ideas of factors, prime factors and prime numbers. It also describes two methods for the prime factorisation of whole numbers.

Introduction: Different types of factors

The **factors** of a number are the whole numbers that are multiplied together to give that number, sometimes the word **divisors** is used instead of factors. Working out the factors of a number has many uses, for instance in the simplification of fractions and factorisation of algebraic expressions (see study guides: *Cancelling Down Fractions* and *Simple Factorisation*). For example 1×16 , 2×8 and 4×4 all equal 16. So the factors of 16 are 1, 2, 4, 8 and 16. In mathematics the word **proper** has a specific meaning. The **proper factors** of 16 do not include 16 itself and so are 1, 2, 4, and 8.

The list of factors can often be broken down further into a list which only contains **prime numbers**. These are called the **prime factors** of that number and are usually written *in ascending order*. The method for finding the prime factors of a number is called **prime factorisation**.

Prime numbers

In order to work out the prime factors of a number you need to know whether each factor is a prime number or not. The definition of a prime number, commonly called simply **prime**, is:

a prime number has just two factors, 1 and itself.

The first nine prime numbers are:

2 3 5 7 11 13 17 19 23

Importantly 1 is *not* a prime number as it has only a single factor (1, which is also itself). Just as importantly 2 is the *only* even prime number. This is because all the other even numbers have 2 as a factor, and therefore cannot be prime. A list of all prime numbers less than 1000 is available on the factsheet: *Prime Numbers Under 1000*.

All the whole numbers (apart from 1) which are not prime are called **composite numbers**. Each composite number is 'made' by multiplying together a *unique* list of prime numbers. This fundamental idea is called the **Prime Factorisation Theorem** by mathematicians. It was known as far back as Euclid (fl. 300 BC) and remains true today. In fact, if the prime factorisation theorem was *not* true, none of the strategies in this guide would work.

Strategies for finding prime factors

The most effective way of finding the prime factors of a composite number is to have good knowledge of the prime factors of smaller numbers. This is especially true if the composite number has three or more digits. So a good initial strategy is to concentrate on finding the prime factors of small numbers – ones with one or two digits. It is also important for you to be able to divide a number easily by another number.

All strategies to find prime factors involve breaking down the number in question into multiplications of smaller numbers. Eventually you will be left with only prime numbers multiplied together. There are many ways to do this; this guide will concentrate on two common methods. The first method, **division by ascending prime numbers**, requires you to have good division skills, whereas the second method, **factor trees**, requires a good command of your times tables.

Division by ascending prime numbers

The method of division by ascending prime numbers involves repeated, *valid* division of the number you wish to factorise by the prime numbers in ascending order. A valid division results in a whole number answer, some strategies to tell you whether division will be valid or not are given in the study guide: *Rules for Dividing Whole Numbers*.

Method of division by ascending prime numbers

The method involves dividing the number you have by the smallest possible prime number. This process is repeated on the result until you have a prime number.

Example: Express 36 in its prime factor form.

As 36 is an even number you can divide by 2 (the smallest possible prime number) to give 18. Expressed another way:

$$36 = 2 \times 18$$

What you have done is recognised that 2 is a prime factor, a factor which cannot be broken down any further. However you still have a factor of 18, which is a composite number which needs to be broken down using the same method. As 18 is even you can divide by 2 again to get 9, in other words $18 = 2 \times 9$. Write this underneath as follows:

$$\begin{aligned} 36 &= 2 \times 18 \\ & \qquad 2 \times 9 \end{aligned}$$

Again you have identified a prime factor of 2 but also a composite factor of 9. You now need to prime factorise 9 but as 9 is odd you cannot divide by 2 and so move on to the next prime number which is 3. Write 3×3 beneath 9 in your list:

$$\begin{aligned} 36 &= 2 \times 18 \\ & \qquad 2 \times 9 \\ & \qquad \quad 3 \times 3 \end{aligned}$$

As 3 is a prime number you have finished factorising. All that remains is to write the prime factor form of 36. If you follow the above method, the prime factor form is shown to the left of the list of factors (as circled below).

$$\begin{aligned} 36 &= 2 \times 18 \\ & \qquad 2 \times 9 \\ & \qquad \quad 3 \times 3 \end{aligned}$$

So:

$$36 = 2 \times 2 \times 3 \times 3$$

Example: Express 63 in prime factor form.

As 63 is odd you cannot divide by 2. The next prime number is 3 which does divide 63 to give 21.

$$63 = 3 \times 21$$

21 is also divisible by 3 to give 7. As 3 and 7 are prime numbers you have finished the factorisation so:

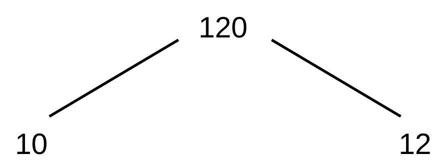
$$\begin{aligned} 63 &= 3 \times 21 \\ & \qquad 3 \times 7 \end{aligned}$$

Which gives $63 = 3 \times 3 \times 7$.

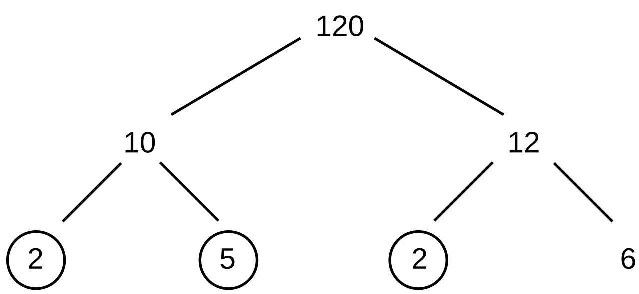
Factor trees

An alternative method for prime factorisation is using a **factor tree**. A factor tree is constructed for a particular number by looking for two factors which multiply together to give that number. These factors are written below the original number. If a factor is prime, then it is circled. Factors which are not prime are broken down in the same way as the original number, until all the factors are prime. Although this method is similar to the first method, it can sometimes prove to be much quicker.

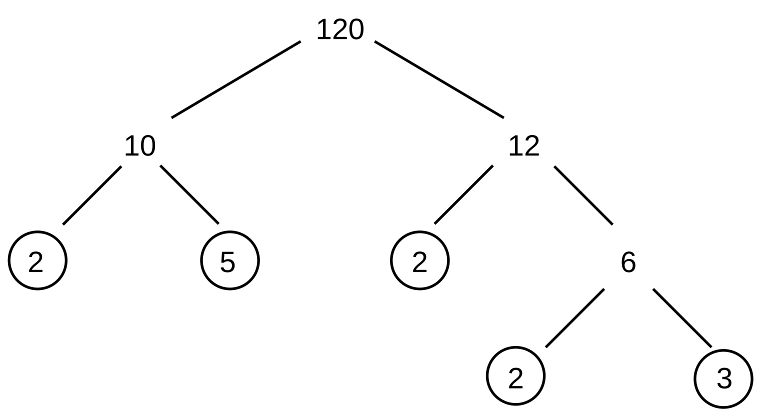
Example: What are the prime factors of 120?



$120 = 10 \times 12$ so 10 and 12 are written below 120.



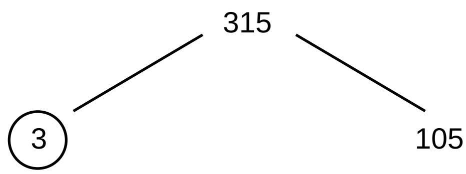
10 and 12 are not prime. As $10 = 2 \times 5$, 2 and 5 are written below 10. Also $12 = 2 \times 6$ and so 2 and 6 are written below 12. The prime numbers 2 and 5 are then circled.



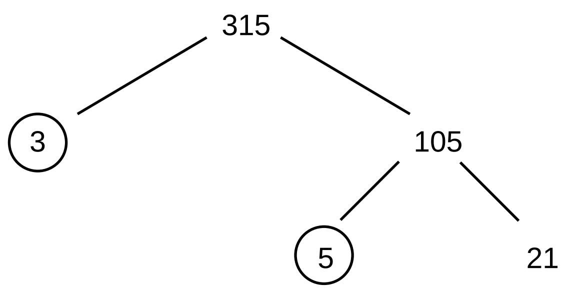
The only number left that is not prime is 6. As $6 = 2 \times 3$, 2 and 3 are written below 6. 2 and 3 are prime and are circled.

The prime factors of 120 are all the circled numbers in the factor tree. So $120 = 2 \times 2 \times 2 \times 3 \times 5$, where the prime factors are written in ascending order. It does not matter if you choose different factors as your starting point (for example, you could have chosen $120 = 2 \times 60$) the answer will be the same.

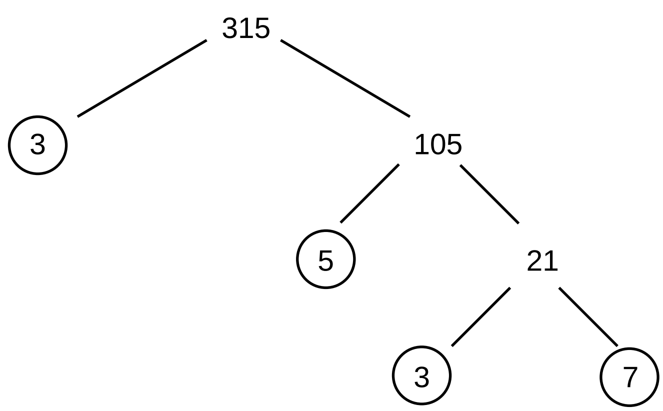
Example: What are the prime factors of 315?



$315 = 3 \times 105$ so 3 and 105 are written below 315. 3 is prime and so is circled.



105 is composite. As $105 = 5 \times 21$, 5 and 21 are written below 105. The prime number 5 is circled.



The only number left that is not prime is 21. As $21 = 3 \times 7$, 3 and 7 are written below 21. 3 and 7 are prime and are circled.

The prime factors of 315 are all the circled numbers in the factor tree. So $315 = 3 \times 3 \times 5 \times 7$.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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