Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Numeracy

Powers of 10 and Standard Form

This guide gives an introduction to powers of 10. It shows how powers of 10 are useful in describing large and small numbers. This shorthand is commonly called standard form.

Positive powers of 10

Multiplying the number ten by itself a certain number of times always results in numbers which begin with 1 and are followed by zeroes (for example 10000). In mathematics there is a shorthand way of representing multiplication of a number by itself known as a **power** (or **index**). The number 10 is a good place to start learning how powers work as the numbers produced when multiplying 10 by itself are easy to calculate. Start with the number 10 and multiply it by itself:

$$10 \times 10 = 100$$

Now multiply 100 by 10, can you see that this is the same as $10 \times 10 \times 10$? So:

$$10 \times 10 \times 10 = 100 \times 10 = 1000$$

There is a pattern emerging: every time you multiply by 10 in this way you write an extra zero at the end of the number. If you do this six times you get one million:

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1000000$$

This would get tedious as the numbers get bigger and bigger. There is a shorthand way of writing these large numbers. A **superscript to the right** of a number (in this case 10) is used to indicate how many times that number has been multiplied by itself. Using the example of one million, you have multiplied 10 by itself six times which can be written using the shorthand as:

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

In mathematics this is referred to as a **power of ten** and is said “ten to the power six”.

A simple way of working out the power of 10 represented by a number beginning with 1 and followed by zeroes is to count the number of zeroes. In other words the number of

zeroes is equal to the number of times 10 is multiplied by itself. So $100 = 10^2$ as 100 is 1 followed by two zeroes, $1000 = 10^3$ and so on. Importantly $10^1 = 10$ and $10^0 = 1$.

Example: Express the number 100000000 as a power of 10.

100000000 is 1 followed by eight zeroes. This is 10^8 expressed as a power of ten which represents 10 multiplied by itself eight times.

Negative powers of 10

Powers of 10 which are negative numbers also have a useful meaning. They represent proper fractions with the numerator of 1 and a denominator of the corresponding positive power of 10. In other words **multiplying by a negative power of 10 is the same as dividing by the corresponding positive power of 10**. Mathematically, this can be written as:

$$\times 10^{-n} = \div 10^{n}$$

Example: What fraction and decimal number does 10^{-6} represent?

To represent 10^{-6} as a fraction you use the rule above with $n = 6$. So:

$$10^{-6} = 1 \times 10^{-6} = 1 \div 10^{6} = \frac{1}{10^6} = \frac{1}{1000000}$$

which is one millionth. One millionth is 0.000001 expressed as a decimal number. You can also find the decimal equivalent of a negative power of 10 by writing the positive power of 10 backwards and putting a decimal point after the first zero.

Numbers in standard form

In disciplines which use mathematics as a foundation (for example science and economics) you may come across very large and very small numbers. For example, the distance across a galaxy is very large and the distance across an atom is very small. It takes many digits to write a very large or very small number. This can lead to human error when writing or keying in such numbers. Every number can be written as a number between 1 and ten multiplied by a power of 10. When you write a number in this way you are expressing it in **standard form.** Mathematically, for a number to be in standard form it must fit the pattern:

$$a \times 10^{n}$$

where *a* is bigger than or equal to 1 but less than 10 and *n* is an integer (a positive or negative whole number or 0). You can work out the general size of a number in standard form by looking at the value of *n*:

> **If *n* is a positive integer, the number in standard form is bigger than or equal to 10.**
>
> **If *n* is a negative integer, the number in standard form is between 0 and 1.**
>
> **If *n* is zero, the number in standard form is bigger than or equal to 1 and less than 10.**

In order to convert a number to standard form you need to separate the number into two parts which are then multiplied together.

Part 1: **The calculation of *a***. The first part to find is a number between 1 and 10. To do this put a decimal point in the number after the first non-zero digit (working from left-to-right). This will always give a valid value for *a*.

Part 2: **The calculation of *n*.** You can calculate *n* by working out the power of 10 required which, when multiplied by *a*, gives your original number.

Example: Write 1456 in standard form.

Part 1: Put a decimal point after the first digit in 1456 to give 1.456, this is *a*.

Part 2: Now you need to deduce the correct power of ten. At the moment you have:

$$1.456 \times ? = 1456$$

Can you see that you have to multiply by 1000 to make the mathematics correct? So $1.456 \times 1000 = 1456$. As 1000 is 10^3, *n* is 3 and you have:

$$1456 = 1.456 \times 10^3$$

which is in standard form.

Example: Write 0.0235 in standard form.

Part 1: First position the decimal point after the first non-zero digit (working from left-to-right) to find *a*, this gives 2.35.

Part 2: At the moment you have:

$$2.35 \times ? = 0.0235$$

It would be more obvious to *divide* 2.35 by a power of 10 (specifically 100) to make 0.0235. As 100 is equal to 10^2 and dividing by 10^2 is identical to multiplying by 10^{-2}, *n* is -2. Therefore you have:

$$0.0235 = 2.35 \div 10^2 = 2.35 \times 10^{-2}$$

which shows that 0.0235 is 2.35×10^{-2} in standard form.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761
Ask: ask.let@uea.ac.uk
Click: https://portal.uea.ac.uk/student-support-service/learning-enhancement

There are many other resources to help you with your studies on our website.
For this topic, these include questions to practise, model solutions and a webcast.

Your comments or suggestions about our resources are very welcome.

	Scan the QR-code with a smartphone app for a webcast of this study guide.	