

Steps into Numeracy

Different Kinds of Numbers

This guide introduces, in an historical perspective, the different types of numbers which mathematicians use today and the various names and symbols which they are given.

All of us first experience the concept of a number when we are very young in relation to counting objects. You can easily start to write them down;

1 2 3 4 5 ...

These numbers are usually called the **Natural Numbers** by mathematicians, they can also be thought of as the **Counting Numbers** although no definitive convention has been agreed. (They can be thought of as the number of objects in a collection.) These numbers began to be written down in a symbolic way by the Egyptians and Babylonians (c.3000 BCE) mainly as a way of book-keeping. The Egyptians used a system of base-10 (identical to us today) and the Babylonians used base-60 (remnants of which exist in our measures of time and angle, see study guide: [Angles](#)). Many properties of natural numbers were deduced by these people and some astounding records of their achievements still exist today (for example the Rhind Papyrus, c. 1650 BCE).

You may notice that zero is absent from the list of natural numbers and although the Egyptians knew of its existence, zero was not necessary for calculations involving amounts. It was first written in its present form in Cambodia c.683 AD and it was around that time that the great Indian mathematician Brahamagupta wrote of the arithmetical rules of zero. If you include zero in our list of natural numbers you get the **Whole Numbers**;

0 1 2 3 4 ...

Negative numbers were first used in China during the Han Dynasty (202 BCE – 220 AD) but for many years results which were negative were considered absurd. Again it was Brahmagupta who wrote down the rules for their manipulation. If we consider the negative analogues of the natural numbers and add them to our description we get the **Integers**;

... -3 -2 -1 0 1 2 3

The set of all integers is denoted by the symbol \mathbb{Z} from the German word 'zahlen' and contains all the whole numbers.

Before the 'discovery' and acceptance of negative numbers other types of numbers were known. When numbers are divided not all answers are integers (for example 3 divided by 5) and so a new type of number was required, the fractions. Mathematically fractions are known as **Rational Numbers** and strictly they express the ratio of two integers (the denominator cannot be zero). The set of rational numbers is given the symbol \mathbb{Q} as in quotient. Certain rational numbers, those with a numerator of one (unit fractions) and also two-thirds, were known to the Egyptians; also a limited number of unit fractions were used by the Babylonians. A more general treatment evolved in Greece and is encapsulated in Book V of Euclid's *Elements* (c. 300 BCE although no contemporary copies exist). The set of rational numbers contains all the integers along with so-called **terminating decimal** numbers (such as 3.6 and 8.72) and the repeating or recurring decimal numbers (such as a third which is 0.333... continuing on forever). See study guide: [Types of Fractions](#).

The Greeks also knew of another type of number that didn't fit the patterns described so far. To the Greeks if two numbers had a 'common measure' then the ratio of these numbers was a rational number, if not then the number was thought to be **incommensurable**. Many such incommensurables appear in the Greek's exploration of geometry. For example consider the length of the hypotenuse of a right-angled triangle whose other two sides have length of 1. The theorem of Pythagoras tells us that the length is $\sqrt{2}$ which was proved by Hippasus of Metapontum (c. 700 BCE) not to be a rational number. Euclid coined the term **Irrational** for such numbers and they form a set completely apart from the rational numbers. Irrational numbers form non-terminating decimal numbers which *never* repeat themselves.

Many irrational numbers form solutions to algebraic equations with rational coefficients. For example, one of the solutions to the equation $x^2 - 2 = 0$ is $\sqrt{2}$, however there are many irrational numbers which do not fall into this category and these are called **Transcendental Numbers**. Many of the most famous numbers, for example π and e are transcendental. Showing if a number is transcendental or not proved to be difficult and it wasn't until 1844 that Joseph Liouville actually wrote one down; e was shown to be transcendental in 1873 by Charles Hermite and π in 1882 by Ferdinand Lindemann.

Together the rational and irrational numbers make up the set of **Real Numbers** which is given the symbol \mathbb{R} . When you draw a number line you are sketching a visual representation of all the real numbers.

In the Renaissance a solution to the general cubic equation was found by Gerolamo Cardano (1501-1576) which implied the existence of a new type of number. It was thought at the time that square roots of negative numbers were impossible, after all what number multiplied by itself equals a negative number? But it seemed from Cardano's formula that these numbers were necessary in order to complete certain calculations. The book *Algebra* written by Rafael Bombelli (1526-1572) was the first place to define numbers that were neither positive nor negative – the **Complex Numbers**. Complex numbers rely on the premise that the square root of a negative number does exist and defines $\sqrt{-1} = i$ as a building block, for example $\sqrt{-7} = \sqrt{7}\sqrt{-1} = \sqrt{7}i$. This notation was due to Leonhard Euler (1707-1783). Many of the great mathematicians in the eighteenth century worked on complex numbers, and their discovery and acceptance proved revolutionary. A complex number comprises two parts, one that is a purely real number and one that is a real number multiplied by i . If the purely real part is zero then the complex number is called **Imaginary**. Of course if the number multiplied by i is zero then you get a real number. This shows that the set of complex numbers includes the set of the real numbers. The set of complex numbers is given the symbol \mathbb{C} . See study guide: [Basics of Complex Numbers](#).

Today mathematicians consider numbers to be divided into the following sets:

- N the set of positive integers
- Z the set of integers
- Q the set of rational numbers
- R the set of real numbers
- C the set of complex numbers

any number you can think of falls in one of these sets.

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