Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Statistics

Basics of Probability

This guide is an introduction to probability. It will explain the different ways that probabilities can be written both numerically and symbolically. It will also discuss theoretical and experimental probabilities along with independent and mutually exclusive events.

Introduction

You will come across **probabilities** every day, in news articles discussing research, research articles, and many other instances. They may often be referred to as **chance** or **risk**. It is important that you can interpret the probabilities correctly as you may be required to report them to a third party accurately or to write about them yourself. The study of probability is a rich and important branch of mathematics and is fundamental in the development and understanding of **statistics**. Probability was first investigated in the seventeenth century by mathematicians such as Pascal and Huygens to gain a greater understanding of gambling and games of chance.

Quantifying and writing a probability

Probability is the study and quantification of the **likelihood** of **events** happening. Strictly a probability can take any value between 0 and 1. Specifically:

Probability of 0:	Event cannot happen.
Probability bigger than 0 but less than 1:	Event might happen
Probability of 1:	Event is certain to happen.

You can think that, as the numerical value of a probability increases, then the likelihood of the event it describes occurring also increases. Probabilities are also often written as **percentages** and **fractions** (see study guides: *Percentages*, *Types of Fractions* and *Ratio*). Remember decimal numbers, fractions and percentages are interchangeable and it is often the context in which the probability arises which defines the form it takes.

There are conventional ways to write probabilities. For instance, think about asking someone to pick a letter of the alphabet **at random**. You can define an event as someone picking the letter *E* for example. It is important that you are clear that the event is different from the probability – the probability is a number that quantifies the chance of an event happening.

You would write the probability of picking the letter *E* as:

$P(E)$	probability of event *E* happening

The *P* stands for "probability" and *E* stands for the event "picking the letter *E*". In general events are denoted by capital letters.

Types of probability

Investigations into probability fall into two main areas: **theoretical** and **experimental**. Think about randomly choosing a letter from the alphabet, what is the chance that the letter is *E* ? As there are 26 letters in the alphabet and only one of them is *E* then the chance of picking *E* at random is 1 out of 26, this is a **theoretical probability**. By definition:

$$\text{Theoretical probability of event occurring} = \frac{\text{number of ways event can occur}}{\text{total number of possible events}}$$

You can see now why it is common to express probabilities as fractions as this equation takes the form of a fraction. The probability for picking the letter *E* can be written as either:

$$P(E) = \frac{1}{26} \quad \text{or} \quad P(E) = 0.038 \quad \text{or} \quad P(E) = 3.8\%$$

(note that they all represent the same number, 0.038)

You could also investigate the probability of picking an *E* by setting up an experiment to calculate an **experimental probability**. You could make a bag with 26 balls in it (each ball having a different letter on) and ask (say) 100 people to pick a ball from the bag. You could then record how many times the ball with *E* on was picked. By definition:

$$\text{Experimental probability of event occurring} = \frac{\text{number of times event occurs}}{\text{total number of experiments}}$$

Here, if 4 of the 100 people picked the *E* ball then:

$$P(E) = \frac{4}{100} \quad \text{or} \quad P(E) = 0.04 \quad \text{or} \quad P(E) = 4\%$$

(again these all represent the same number)

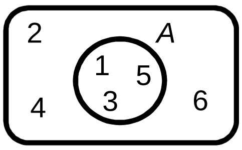
As the number of experiments is increased then the experimental probability should get closer to the theoretical probability. Experimental probabilities are often used when you cannot calculate the theoretical probability as you do not have enough information.

Visualising probability: Venn diagrams

A picture can often help immensely in understanding and explaining the concepts of probability. Along with **tables**, the most common way of visualising probability is the **Venn diagram**. (Venn diagrams are also common in the study of **logic** and **sets**, see study guide: *Operations on Sets.*) A Venn diagram consists of a rectangle (called the **universe**) around a number of appropriately labelled interlocking shapes which represent outcomes of an event. For representations of 1, 2 and 3 events these shapes are usually circles, for higher numbers of events the shapes become more elaborate.

Example: Draw the Venn diagram that describes the event of throwing an odd number on a normal six sided die.

Firstly you define the event "$A =$ Throwing an odd number", you must be clear that this is the event itself and not the theoretical probability of it happening, written $P(A)$.



You can see that the odd numbers are contained within the circle labelled A and the even numbers are outside this shape but within the universe.

You can see that 3 of the total six events are inside A and so:

$$P(A) = \frac{\text{Number of ways of throwing an odd number}}{\text{Total number of ways}} = \frac{3}{6} = \frac{1}{2} = 0.5\,.$$

Complement

Looking at the Venn diagram in the previous section you can see that three of the six events are **not** in A. In set theory, events that are not in A form the **complement** of A, this is often written as $\overline{A}$ or A^c (this guide will use the first symbol). You can write the probability of the complement of A happening as $P(\overline{A})$.

So, taking A as throwing an odd number, $P(\overline{A}) = \frac{3}{6} = \frac{1}{2} = 0.5$

In probability A and $\overline{A}$ are called **complimentary events** which says that *all* events are either in A or not in A. Because of this you can write a neat piece of mathematics:

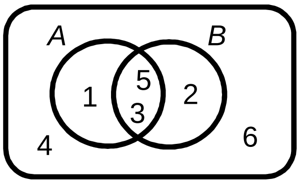
$P(A)+P(\bar{A})=1$ **Law of complementary events**

Probabilities involving more than one event: "and"

Naturally probability is not just about whether or not one event occurs, it can help you understand situations where multiple events can happen. Let's think about another event that can happen when throwing a six sided die:

$B=$ Throwing a prime number (that's 2, 3 or 5)

You can draw a Venn diagram which includes the events *A* and *B*.



Here you can see that 1 is odd and not prime, 2 is prime and not odd, 3 and 5 are both odd **and** prime, and 4 and 6 are neither odd nor prime.

The overlap of the two circles has a specific meaning in set theory and is called the **intersection** of *A* and *B*, written $A \cap B$ (see study guide: *Operations on Sets*). You can see that you can write the corresponding probability which describes the chance of two events occurring at the same time (throwing a number which is both odd and prime):

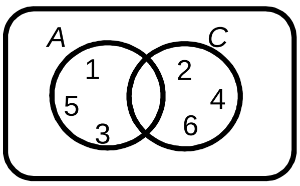
$P(A \cap B)=\frac{2}{6}=\frac{1}{3}=0.\dot{3}$ Probability of *A* and *B* occurring **at the same time**.

Mutually exclusive events

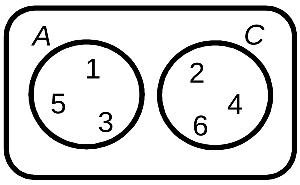
Now let's consider a third event:

$C=$ Throwing an even number

A Venn diagram showing *A* and *C* can be drawn in two ways here as there are no numbers which are both odd and even.



or



Two or more events that cannot occur at the same time are called **mutually exclusive events**.

For example being a cat and being a dog are mutually exclusive events, being in Ipswich and being in Norwich are mutually exclusive events and so on. Here you can write "*A* and *C* are mutually exclusive events". In other words throwing an odd number and throwing an even number are mutually exclusive events and so:

$$P(A \cap C) = 0$$

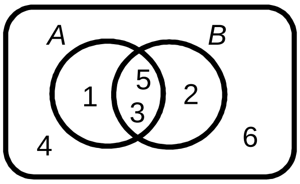
This is true for all mutually exclusive events.

It is important that you do not confuse mutually exclusive events with independent events which are completely different and are discussed in the study guide: *More Complicated Probability*.

Probabilities involving more than one event: "or"

Probability can also help you understand outcomes that are described by either one event or another or both. Returning to events *A* and *B*:

$A =$ Throwing an odd number
$B =$ Throwing a prime number



You can see that:

throwing a 1 is only described by event *A*
throwing a 2 is only described by event *B*.
throwing a 3 (or a 5) is described by both events *A and B* as 3 (and 5) is both odd and prime.

So throwing a 1, 2, 3, or 5 is described by either event *A or* event *B or* both event *A* and *B*. This is a special case in probability and is written as $P(A \cup B)$. In set theory this is called the **union** of *A* and *B*. You can see that:

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3} = 0.\dot{6}$$ Probability of *A* or *B* or both occurring.

If you look at the Venn diagram containing *A* and *B* at the beginning of this section you can see that the probability of $A \cup B$ occurring is the probability of *A* (throwing 1, 3 or 5) plus the probability of *B* (throwing 2, 3 or 5) and then subtracting the probability of those outcomes described by $A \cap B$ (throwing a 3 or a 5). You subtract this probability as throwing a 3 or a 5 is described by both *A* and *B* and so technically you have counted them twice. You can write this reasoning mathematically which gives a very useful result linking the probabilities of $A \cup B$ and $A \cap B$:

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)} \quad \text{or} \quad \boxed{P(A \cap B) = P(A) + P(B) - P(A \cup B)}$$

Example: Use the left-hand equation above to calculate the probability of $A \cup B$.

From the above discussions you have seen that $P(A) = \frac{3}{6}$, $P(B) = \frac{3}{6}$ and $P(A \cap B) = \frac{2}{6}$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6}$$

as deduced above.

Remember that the probability of mutually exclusive events occurring at the same time is zero. So now let's consider events *A* and *C* where $P(A \cap C) = 0$. This means that:

$$\boxed{P(A \cup C) = P(A) + P(C)}$$

Law for mutually exclusive events.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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	Ask:	ask.let@uea.ac.uk
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